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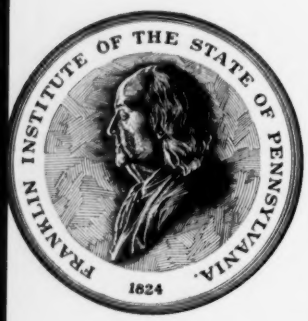

1954

Philadelphia Leaders in Agriculture
STEVENSON W. FLETCHER

The Radiation Green's Functions
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PARRY MOON AND DOMINA EBERLE SPENCER

Franklin Institute:
Research Notes
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VOL. 258 NO. 2

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PHILADELPHIA LEADERS IN AGRICULTURE *

BY

STEVENSON W. FLETCHER [1]

During the past few months I have been pleasurably engaged in gathering material for a projected book to be entitled "Leaders in Pennsylvania Agriculture." An attempt will be made to appraise the respective contributions of those who have done most to advance Pennsylvania agriculture. In the fallible judgment of one man, there are thirty-eight such leaders. Sixteen of these, or nearly half, were Philadelphians. The agriculture of Pennsylvania is indebted to Philadelphia for a large proportion of the leadership that has brought it to its present high standard of achievement.

WILLIAM PENN, 1644–1718

First of those who helped to mould Pennsylvania agriculture was the founder of the Commonwealth. William Penn resided in his woodland only four years but he made a permanent impression on its agriculture. His gardens at Pennsbury were an inspiration and object lesson to those who lived in or near his "fine, greene Country Towne." The national prominence in ornamental horticulture of Philadelphia in later years, particularly the large number of beautiful country seats of the wealthy, reflected the horticultural taste of the founder.

William Penn was not a successful farmer. His vineyard of European wine grapes on the banks of the Schuylkill was a dismal failure and his venture into milling was no more successful. The main contribution of William Penn to Pennsylvania agriculture was his democratic policy in the sale of land. In the Great Britain of his time most

* A Lecture presented at the Joint Meeting of The Franklin Institute and the Philadelphia Society for Promoting Agriculture, in the Lecture Hall at The Franklin Institute, March 17, 1954.

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farmers were tenants, not owners. Most land was held in great feudal estates. Land ownership carried with it economic, social and political advantages which were forever denied to most of those who tilled the land. It was the desire of William Penn, on the other hand, to found a commonwealth of small, independent land owners. Before coming to America he sold a few large tracts, the so-called "Baronies," and he reserved a number of proprietary tenths, or "Manors," but these were such in name only. The great majority of sales was to small, impecunious freeholders. In all of Penn's advertisements of his Province in the countries of eastern Europe he stressed the opportunity it offered for a poor man to own a farm. Most of his sales were in lots of 100 to 500 acres. Unlike some other American colonies, where large tracts were sold or granted to a few favored individuals, small one-family farms have prevailed in Pennsylvania from the beginning. It was this democratic land policy of William Penn, no less than his charter of civil and religious liberty, that made Pennsylvania the haven of the oppressed and homeless of all nations.

JOHN BARTRAM, 1699–1777

The first indigenous leader of Pennsylvania agriculture was John Bartram. This gentle, self-educated Quaker has a secure place in the American Hall of Fame as a pioneer botanist, but he was an outstanding agricultural figure as well. Farming was his main means of livelihood. All of his early explorations were undertaken at his own expense; the modest sums that he received in later years for plants and seeds shipped to Europe merely supplemented his major source of income, his farm.

The gardens of America and Europe were enriched by the wealth of plant material collected by John Bartram in his journeys to the frontiers. Spurred on, not by hope of gain, but by an unquenchable curiosity about nature, he climbed the highest mountains, forded the swiftest rivers, endured severe privations and risked his life in territory inhabited only by wild beasts and hostile Indians. Neither difficulty nor danger deterred him. Truly he was an heroic figure.

John Bartram exerted a powerful influence for the improvement of agriculture. At a time when the average yield of wheat near Philadelphia was about ten bushels to the acre he commonly secured thirty or more from his 300-acre farm on the west bank of the Schuylkill. He maintained the fertility of his farm with manure composts and by sowing red clover long before that soil-improving crop had come into common use. He was the moving spirit in organizing the first incorporated drainage district in America. In cooperation with neighbors, several thousand acres of rich marsh land along the Schuylkill were reclaimed with dikes, flood gates and ditches. "The expense," he said, "is very considerable, but such is the goodness of the grass

for fattening cattle that the produce of three years pays all advances." So successful was this enterprise that by 1800 many square miles of marsh land along the Delaware and its tributaries had been brought under cultivation.

When John Bartram began his hazardous journeys into the wilderness he walked alone. His friends considered these expeditions foolhardy and futile. He said, "I know not a solitary one who would walk a mile with me." Today, many would be proud to walk with this great Patriarch of American botany, even for but a mile.

BENJAMIN FRANKLIN, 1706–1790

The many-sided Franklin—shrewd business man, inventor, scientist, diplomat, statesman—was without a peer in the variety and significance of his achievements. City born and bred, his interests were mainly urban, not rural, yet he should be numbered among the great leaders of Pennsylvania agriculture.

Franklin's contribution to agriculture was as a statesman and man of affairs rather than as a farmer. He owned a 200-acre farm near Kembleville, Chester County, but there is no evidence that he operated it personally. For many years it was thought that he owned a farm near Burlington, New Jersey but recent investigations by Dr. Carl R. Woodward of Rutgers University have shown that this was not the case.

Franklin's name became a household word throughout the colonies through his publication of *Poor Richard's Almanack* for twenty-five years, from 1733 to 1758. At that time books and journals on agriculture were practically unknown in America. The Bible and the almanac were about the only printed sources of authority in farm homes, the one on spiritual, the other on temporal matters. Poor Richard's maxims were intended to inculcate the ideals of thrift and honesty and many were drawn from farm life. Such pithy counsel as:

> Plough deep, while Sluggards sleep
> And you shall have Corn to sell and keep

and

> He that by the Plough would thrive
> Himself must either hold or drive

had a profound influence on the thinking and practice of farmers.

Throughout his life, especially while traveling through the colonies as Postmaster-General and during his sixteen years of diplomatic service in Europe, Franklin had an observing eye for efficient farm practices; these he passed on to his fellow countrymen. He facilitated the exchange of European and American crops, seeds and plants through his friend John Bartram and others.

The major contribution of Benjamin Franklin to agriculture, however, was his initiative in promoting organizations for agricultural education and research. In 1743 he organized the Society for the Promotion of Useful Knowledge, which later became the American Philosophical Society. Among the objectives of this society, as stated by Franklin, were: "To investigate all new discovered plants, herbs, trees. . . the nature of soils. . . improving the breeds of livestock. . . ." It had a committee of Husbandry and Rural Improvements which offered premiums to stimulate the development of improvements in farming. John Beale Bordley was a member of this committee and perhaps derived from it inspiration to found the Philadelphia Society for Promoting Agriculture.

Franklin was first in America to advocate the instruction of youth in the science and practice of agriculture. In his "Proposals Relating to the Education of Youth," in 1759, he stated, "While they are reading Natural History, might not a little Gardening, Planting and Grafting be taught?" This proposal resulted in the founding of Philadelphia Academy, forerunner of the University of Pennsylvania. The founding of the Farmers' High School, now The Pennsylvania State University, in 1855, may be traced directly to the influence of Franklin. Its chief sponsor was the State Agricultural Society, which was the child of the Philadelphia Society for Promoting Agriculture which, in turn, grew out of the American Philosophical Society. Franklin set in motion currents of agricultural education that continue to serve Pennsylvania agriculture to this day.

JOHN BEALE BORDLEY, 1727–1804

From 1785 to 1860 practically all progress in Pennsylvania agriculture resulted from the activities of gentlemen farmers, most of them members of the Philadelphia Society for Promoting Agriculture, organized in 1785. Eight of the nine leaders during this period were members of this time-honored organization.

The dominant motive of these well-to-do patrons of agriculture was patriotism. The triumphant outcome of the Revolution released pent-up energies in the young republic. In pride of achievement, the patriots sought to make their nation more prosperous. First to receive attention was its major industry, agriculture. This was the occupation of more than ninety per cent of the population and the chief source of the national wealth, hence it was a matter of concern to men in all walks of life.

At a time when most working farmers were indifferent to if not openly scornful of science as a possible aid to farm practice, this little band of gentlemen farmers conducted experiments and tested new methods developed at home and abroad. They were imbued with the spirit of inquiry. Always they were the butt of the jibes of working

farmers. Sneered one, "They are mostly gentlemen who have acquired fortunes at mercantile or other lucrative pursuits and who, wearied with the dull round of money getting, have determined to improve agriculture by money spending." But improve it they did. In time, the results of their observations and experiments seeped down to the grass roots of every-day farming.

First and foremost of the gentlemen farmers was John Beale Bordley. After 1770 this wealthy Baltimore lawyer and Judge of the Admiralty withdrew from public life and retired to his 1600-acre farm on Wye Island, Chesapeake Bay. His farming was in the patriarchal manner, closely resembling in organization the great plantations of the South. The many and diversified interests of the farm made it, in effect, a self-sufficient rural village in which raw materials from crops and livestock were manufactured into food and clothing for the considerable number of inhabitants of his domain.

Judge Bordley was a student of the sciences that underlie agriculture and was well acquainted, through travel and reading, with the marked improvements in British agriculture resulting from the work of Charles Townshend, Arthur Young and other British leaders. He was not content to follow the beaten path but struck out into new fields. He attempted, through experimentation, to ascertain the principles on which successful farm practice rests. His farm was a veritable agricultural experiment station. Judge Bordley struck the first clear note of scientific agriculture in America. The results of his experiments were published, in 1799, in his celebrated *Essays and Notes on Husbandry and Rural Affairs* which was chiefly responsible for bringing in "The New Agriculture."

Judge Bordley has been called "The Father of Crop Rotations in America." He was not the originator of this epochal practice but was its most influential advocate in America. It was based on the so-called "Norfolk System" of crop husbandry developed in England by Charles Townshend and others. On his Wye Island farm Judge Bordley sought to adapt the English course of crops to American conditions. His book, *A Sketch on Rotations of Crops and Other Rural Matters*, published in 1792, found wide acceptance; soil-improving crop rotations, including grass and clover sods, soon superseded the prevailing all-grain farming. The basis of the standard crop rotations on Pennsylvania farms today was laid by this great experimenter.

Philadelphia was the center of the social, intellectual and political life of Judge Bordley while operating his Wye Island farm; after 1791 he resided here. In 1783 he was elected a member of the American Philosophical Society. In that learned body he found a number of kindred spirits who joined with him in organizing the Philadelphia Society for Promoting Agriculture, in 1785. This is the oldest and,

as measured by the value of its contributions, the most distinguished agricultural society in America.

Judge Bordley was a valiant soul. During his latter years he suffered from a painful and incurable disease. His daughter relates, "Even when under the most severe pain, oftimes so excruciating that his family would entreat him to take rest, he would resume his pen again and again." It was under such trying circumstances that his famous *Essays* was prepared for publication. So passed one of the greatest leaders in early American agriculture.

GEORGE MORGAN, 1743–1810

This eminent Philadelphia business man was an acute observer in agriculture. In his early years he was a leader in the development of the western frontier. He was party to several ambitious colonization projects in the Illinois country, the Indiana country and in Spanish Louisiana. All proved disappointing. In 1779 he moved to "Prospect," his 300-acre farm near Princeton, New Jersey, where he resided for seventeen years. Those were his most productive years; frustrated in his business and colonization ventures, he was successful and honored there.

George Morgan had the insatiable curiosity of the true scientist. His study of the life history of the Hessian fly is classic. About 1785 this imported insect pest began to devastate wheat fields in his neighborhood and threatened to ruin the wheat industry, which then was the mainstay of Pennsylvania agriculture. He made accurate observations on the life cycle of the insect, through its egg, chrysalis and adult stages, and recorded the habits and movements of the fly. From these observations he was able to recommend control measures which are effective to this day. His experiments on crop rotations were exceeded in value only by those of John Beale Bordley and George Logan. In 1785 he received the first gold medal awarded by the Philadelphia Society for Promoting Agriculture for his plan of an efficient barnyard. His change of residence to Morganza, Washington County, Pa. in 1796 removed from the Philadelphia scene one of the most versatile of early American leaders in Agriculture.

GEORGE LOGAN, 1753–1821

This brilliant grandson of the illustrious James Logan, counsellor of William Penn, shares with John Beale Bordley the distinction of being the chief promoters in America of effective methods of maintaining soil fertility, especially through crop rotations. After devoting several years of his young manhood to medical studies and travel in Europe, he returned to "Stenton," his ancestral estate near Philadelphia, and devoted himself to farming and public service. Stenton became one of the most productive farms in America. Thomas Jefferson

appraised him as "the best farmer in Pennsylvania, in both theory and practice."

George Logan was an experimenter of rare attainments. In 1791 he published "Results of a Course of Agricultural Experiments with a View to Discover the Best Rotation of Crops." Like Judge Bordley, he recommended an eight-year rotation in which, he said, "clover is absolutely necessary." He was one of the first to stress the importance of maintaining a high organic content of the soil through frequent plowing down of sods and periodic applications of manure.

The public life of this peace-loving Quaker and United States Senator was fraught with bitter controversy. In 1789 he went to France at his own expense and on his own responsibility to try to avert the threatened war between France and America. He succeeded, almost single-handed. His action was prompted by motives of the purest patriotism, as well as by his Quaker pacifist principles, yet he was branded as a traitor by the jingo press. Said one Philadelphia newspaper, "No sensible man can hesitate to suspect that his infernal design is the introduction of a French army to reorganize our government through the operation of the bayonet and guillotine." A similar mission to England failed to avert the War of 1812. Disheartened by this rebuff, he retired from public life and spent the remainder of his days at his beloved Stenton. George Logan was a towering figure in early American agriculture and well deserves the grateful recognition we give him today.

BERNARD M'MAHON, 1775–1816

This by-product of an unsuccessful rebellion against British rule in Ireland came to Philadelphia as a refugee in 1796. In 1811 he established a nursery and seed farm near the Germantown pike, three miles from Philadelphia. There he propagated the largest number of ornamentals of his day, giving particular attention to rare plants, native and imported.

M'Mahon also operated a seed store at 39 South Second Street, below Market. This became the customary meeting place of Darlington, Nutall and other noted botanists. While M'Mahon participated in the discussions, his wife, a jolly daughter of the Auld Sod, with a rich Irish brogue, officiated behind the counter. It is said that the Lewis and Clark expedition to explore the Northwest was planned there.

In 1806 M'Mahon published *The American Gardener's Calendar;* this was the first serviceable American book on horticulture. It covered all branches of gardening but gave particular attention to ornamentals. M'Mahon was the first American landscape gardener. He was a convincing champion of the American or naturalistic style as distinct from the formal style that then prevailed. His book was the standard authority on horticulture for half a century.

In 1830 a committee of the Pennsylvania Horticultural Society reported, "We owe as much to the late Mr. M'Mahon, as a horticulturist, as to any other individual in America." This was a fair appraisal. M'Mahon's memory is kept green in Pennsylvania gardens by a west coast evergreen barberry which his friend Nutall named *Mahonia* in his honor.

RICHARD PETERS, 1744–1828

Any appraisal of agricultural leaders during the first quarter of the nineteenth century must give preeminence to Richard Peters. His inquiring mind shed light on a wide range of agricultural subjects and for a quarter of a century he guided the society that was most productive in this field.

Richard Peters was a gentleman farmer of means and public distinction. During the Revolution he served as Secretary of the Board of War and he was Judge of the United States District Court of Pennsylvania for many years. "Belmont," his ancestral estate on the banks of the Schuylkill, was one of the show places of Philadelphia. Here gathered George Washington, Thomas Jefferson and many other eminent men to enjoy the sparkling wit of their host and to admire his excellent farming. Belmont, like the farms of Bordley and Logan, was devoted largely to agricultural experiments, which were viewed by many.

Judge Peters made an important contribution to the solution of the foremost agricultural problem of his day—the maintenance of soil fertility. At that time the average yield of wheat in the Philadelphia area had fallen to seven bushels per acre and many thousands of acres of once productive land were abandoned as "worn out." In 1787 Judge Peters delivered a notable address before the Blockley and Merion Society for the Improvement of Agriculture and Rural Affairs on the subject, "The Improvement of Worn-out Soils." This address marked the beginning of progress in this respect.

The most widely publicized contribution of Judge Peters was his discovery of the value of gypsum, or land plaster, as a means of securing luxuriant crops, especially of red clover. For nearly a quarter of a century "clover and plaister husbandry," as it was called, transformed many thousands of acres of impoverished land in southeastern Pennsylvania into productive farms.

In 1799 Richard Peters collaborated with the aged John Beale Bordley in developing a comprehensive plan for fostering agricultural education and research. It provided for the organization of a state agricultural society, county agricultural societies, "Pattern Farms" or experiment stations, instruction in agriculture in public schools and the endowment of professorships in agriculture at colleges. The General Assembly turned a deaf ear to the proposal at that time but eventually all of these objectives were realized.

After the death of Judge Bordley, in 1805, Richard Peters became president of the Philadelphia Society for Promoting Agriculture of which he was a charter member. For twenty-three years he was the spark-plug of this organization. During this quarter century it exercised national leadership in agriculture. Its *Memoirs*, edited by Richard Peters, are the most valuable single repository of information on early American agriculture.

In 1823, when nearing the end of his earthly pilgrimage, Richard Peters reminisced about the early struggles of this pioneer agricultural society: "Through a gloomy period of apathy we kept alive the fire on the altar of our devotion to the great and leading interest of our country. Many individuals and societies have lit their torches at our constant if not always lambent flame, and the brighter they burn the more they contribute to our most sincere satisfaction. We cast our bread upon the waters and now, after many years, we are gathering it in."

So he was gathered unto his fathers, full of years and honors.

WILLIAM COXE, 1762–1831

Philadelphia-born William Coxe was the first great authority on fruit growing in America. Like other agricultural leaders of his time, he was a gentleman of considerable means and of assured social position. After a brief experience in mercantile life he moved to Burlington, New Jersey in order to devote his time almost exclusively to his absorbing interest. He was a student of the arts and sciences and was prominent in the literary and scientific life of Philadelphia.

In 1789, soon after he moved to Burlington, William Coxe laid out his famous experiment plots to test the comparative value of different methods of orchard curlture. His ingenuity in planning these experiments and the care with which he recorded the results would be a credit to any horticultural scientist of today. In addition, he planted all the varieties of tree fruits, domestic and foreign, that he could get.

He first reported the results of his experiments in a paper before the Philadelphia Society for Promoting Agriculture, in 1808. In 1817 he published *A View on the Cultivation of Fruit Trees*, the first American book on fruit growing. It was the standard authority on the subject for nearly half a century. William Coxe has been called "The Father of American Pomology." He well deserves that distinction.

JOHN HARE POWEL, 1786–1856

This very able gentleman farmer was the stormy petrel of Pennsylvania agriculture. He was constantly embroiled in controversy. Born to wealth and high social position, he received a classical education and traveled extensively in Europe before engaging in business in Philadelphia. In 1814 he retired to his country seat, "Powelton," now part of West Philadelphia.

Colonel Powel's chief claim to agricultural fame was as an importer and breeder of superior English breeds of livestock. He was the first to import Durham Shorthorns, a high milking strain of that noted dual purpose breed, in 1822. His dispersal sales of pure bred animals were an important factor in up-grading the cattle of Pennsylvania and neighboring states. In 1825 he made one of the earliest importations of Southdown sheep, which soon superseded the more inferior common stock.

The "battle of the breeds" raged fully as fiercely then as now. Colonel Powel was constantly engaged in acrimonious dispute with Timothy Pickering and others as to the relative value for milking purposes of his Durhams and the Channel Island breeds, Jersey and Guernsey. "So rich is the milk of my cows," he asserted, "that I have produced perfect butter from cream simply by the friction of a spoon in a saucer, and in less than ten seconds." Not being able to match this allegation, his competitors retired from the fray, discomfitted.

In spite of his bombast, Colonel Powel was a highly efficient farmer. At a time when most farmers barely skimmed the surface in plowing, rarely more than four inches, he commonly plowed ten inches. In 1823 he harvested the enormous quantity of 1634 bushels of mangel wurtzels from a little more than an acre. He wrote voluminously on a great variety of farm topics and wielded a powerful influence for improved agriculture.

It is unfortunate that this admirable record of achievement of Colonel Powel was marred by his predisposition to controversy. In 1822 he induced a number of members of the Philadelphia Society for Promoting Agriculture to secede and form a new organization. This soon perished from internal dissension, but the parent society was disrupted for a decade. Thus this brilliant but self-willed man left a record of mixed good and ill, as do all of us in some measure.

NICHOLAS BIDDLE, 1786–1844

The fame of this distinguished Philadelphian rests far more on his outstanding service as a financier, statesman and man of letters than on his contributions as a gentleman farmer, yet he should be numbered among the leaders of Pennsylvania agriculture. He found grateful solace from the harsh criticisms of public life on his country estate, "Andalusia," in Bucks County. When he assumed the presidency of the Philadelphia Society for Promoting Agriculture, in 1838, he found it torn by dissension, largely because of the explosive personality of John Hare Powel. Only an outstanding national figure like Nicholas Biddle could have drawn the warring elements together and so made possible the very productive period that followed

In 1840 Nicholas Biddle imported the first cow known to have been a pure bred Guernsey to land on our shores. His Guernsey herd was the pride of southeastern Pennsylvania. His major contribution to agriculture, however, was in farm economics, not farm practice. He constantly warned farmers against investing their earnings in speculative ventures. He said, "A farmer who has made money often buys more land or shares in a gold mine, when he should buy more livestock to make more manure. Rely upon it, the richest mine for him is his own barnyard." He promoted actively manufactures as a means of expanding the markets for farm produce.

Nicholas Biddle sought to dignify the vocation of agriculture and to enhance the standing of farmers as citizens. No one has expressed this more eloquently than he did in 1822: "The American farmer is the exclusive, absolute, uncontrolled proprietor of the land that he tills. His tenure is not from the government; the government derives its just powers from him. There is above him nothing but God and the laws which he, himself, helped to create. It may not be foretold to what dangers this country is destined when its swelling population, its expanding territory and its complicated interests shall awaken the latent passions of men and reveal the vulnerable points in our institutions. But, whenever these perils come, its most steadfast security, its unfailing reliance, will be the men of the soil." This address was a powerful expression of the achievements, weaknesses and needs of American agriculture. It was widely disseminated throughout the nation and called forth the commendation of three Presidents of the republic—John Adams, Thomas Jefferson and James Monroe.

With the death of Nicholas Biddle, in 1844, there passed from the Pennsylvania scene a powerful national figure, a great scholar and an influential patron of agriculture.

JAMES GOWEN, 1820–1872

The last of the country gentleman leaders of Pennsylvania agriculture was James Gowen. By 1860 new winds were blowing across the agricultural horizon. Working farmers were beginning to emerge as leaders in their vocation and scientists were beginning to make important contributions through research. It was the dawn of a new era.

James Gowen was a well-to-do Philadelphia merchant who, as he expressed it, "quit the bustling scenes of business for the quiet and more respectable station of a farmer." His 120-acre farm at Mount Airy was one of the most productive in the nation; it won the premium awarded by the Philadelphia Society for Promoting Agriculture "for the best managed farm." Like John Hare Powel, he was a prominent importer and breeder of Durham Shorthorn cattle. In 1840 his Durham cow, Dairy Maid, yielded an average of thirty-three quarts

of milk a day four months from freshening. This was the record yield at that time.

In 1847 James Gowen established the Mount Airy Agricultural College on his farm. This was the first agricultural school in Pennsylvania, preceding by eight years the founding of the Farmers' High School. It was not a college but an elementary vocational school. After operating in a desultory way for eight years, it closed because of lack of sufficient enrollment.

James Gowen was chairman of the committee of the Philadelphia Society for Promoting Agriculture which prepared an "Address to the Farmers of Pennsylvania," in 1850, calling a convention to organize a state agricultural society. The convention met in January, 1851 and organized the society. Thus, after the lapse of fifty-seven years, the dreams of John Beale Bordley and Richard Peters were realized.

James Gowen was a member of the first board of trustees of the Farmers' High School, now The Pennsylvania State University, which was sponsored by the Philadelphia Society for Promoting Agriculture and the State Agricultural Society, jointly. He served well in his day and generation.

DAVID LANDRETH, 1802-1880

Pennsylvania horticulture is indebted to three British-trained apprentice gardeners—David Landreth, Robert Buist and Thomas Meehan. David Landreth Sr. emigrated to Philadelphia in 1784 and established a nursery and seed business on High Street, now Market Street. This was the first seed house of importance in America.

David Landreth, Jr. succeeded to his father's business in 1828. Needing more land, in 1847 he moved to a 375-acre farm near Bristol, Bucks County. There he developed the largest seed farm, vegetable trial grounds and arboretum in the world. Landreth seeds were planted in every state and in many foreign countries. Landreth specialized in camellias, rhododendrons and magnolias. From 1825 until 1875 the camellia was the most popular greenhouse flower in Pennsylvania, far more so than the rose.

In 1811 the Landreth firm introduced the first truly white variety of potato; previous varieties had been yellowish. In 1820 it introduced the tomato to garden cultivation. David Landreth was one of the founders of the Philadelphia Horticultural Society. The city of Philadelphia is largely indebted to him for the early development of its horticultural interests.

ROBERT BUIST, 1805-1880

This Scotch apprentice gardener emigrated to Philadelphia in 1828 and established a flower and seed store on Chestnut Street, with a nursery and greenhouses at Twelfth and Lombard Streets. He imported many rare and noteworthy plants but was famous throughout

the nation for his camellias. We are indebted to him for the verbena as a bedding plant and for the *Poinsetta*, which he introduced from Mexico and popularized for the Christmas florist trade.

Robert Buist was a prolific writer. His *American Flower Garden Directory* (1832) was the first useful American book of flower growing and was standard for many years. His *Rose Manual* (1844) and *Family Kitchen Garden* (1847) also were widely used. He came to America near the beginning of the period when Philadelphia was becoming famous for its beautiful gardens and estates. Robert Buist played a prominent role in this development.

THOMAS MEEHAN, 1826–1901

The last of British gardeners to greatly advance Pennsylvania horticulture was Thomas Meehan. Like John Bartram, this brilliant son of an Irish gardener was largely self-educated. "My mother," he said, "was my only school teacher and my only books were the Bible, the Book of Common Prayer and Bunyan's *Pilgrim's Progress*." He acquired a working knowledge of Latin, Greek, French and the natural sciences by studying at home. After serving an apprenticeship at Kew Gardens, he emigrated to Philadelphia in 1848 and established a highly successful nursery of ornamentals at Germantown.

Thomas Meehan was the foremost American vegetable morphologist of his day, especially in the application of science in horticultural practice. He contributed many scientific papers to the proceedings of the Academy of Natural Sciences of Philadelphia, the American Philosophical Society, the American Association for the Advancement of Science and other learned societies at home and abroad. He was editor of *Gardener's Monthly* for the entire twenty-eight years of its existence; this was the most widely circulated horticultural journal in the world next to *Gardeners' Chronicle* of London.

Thomas Meehan raised a potent voice against the wanton destruction of Pennsylvania forests by lumbermen and by fire. He shared with Professor William A. Buckhout of The Pennsylvania State College the distinction of being one of the foremost advocates of forest conservation before the coming of John T. Rothrock. As a member of the City Council of Philadelphia for nineteen years he was chiefly instrumental in adding twenty-three small parks to the city park system. One of these was Bartram's Garden, where he had worked as a young man. No one did more to further the development of Pennsylvania horticulture than Thomas Meehan.

LEONARD PEARSON, 1868–1909

The last two leaders of Pennsylvania agriculture from Philadelphia were college-trained scientists, forerunners of the many professional men who achieved this distinction in the years that followed.

The great dairy industry of Pennsylvania today, the chief source of her agricultural wealth, was made possible largely through the service of Dr. Leonard Pearson. After graduating from the School of Veterinary Medicine of the University of Pennsylvania in 1890, he studied in Germany. While there he became interested in a recently developed method of using tuberculin as a diagnostic agent for the discovery of the presence of tuberculosis in cattle before any physical signs of the disease were apparent.

The first use of the tuberculin test in America was by Dr. Pearson, then on the faculty of the School of Veterinary Medicine, in March, 1892 on a herd of registered Jerseys owned by Joseph E. Gillingham of Villa Nova, Pa. Fifty-one of the seventy-nine animals reacted. The owner generously consented to the slaughter of the reactors in order that a *post mortem* check might be made on the reliability of the test.

This *post mortem* was an epochal event in American agriculture. Gathered around Dr. Pearson were many keenly interested men—farmers, doctors, veterinarians, health officers, representatives of the press. Most of them, even the veterinarians, were skeptical if not openly hostile to the test. The central figure of the occasion, Dr. Pearson, was a beardless youth of twenty-four; he looked like a schoolboy. All the reactors were in prime flesh; nobody could possibly suspect from looking at them that they harbored the dread disease. Yet tuberculosis lesions were found in every one of the fifty-one reactors. The triumph of young Dr. Pearson was complete.

Succeeding tests indicated that nearly one quarter of the 2,000,000 cattle in the state were infected. This caused great concern, not only to farmers, who had lost many animals from the disease annually, but also to the consuming public, for the danger of communicating tuberculosis to humans by using the milk or meat of diseased animals was beginning to be understood. In 1895 the Pennsylvania Livestock Sanitary Board was established by the General Assembly, largely through the efforts of Leonard Pearson, and he was appointed the first State Veterinarian. During the next few years he developed the "Pennsylvania Plan for the Control of Bovine Tuberculosis"; this became world famous and was adopted by almost every other state. By the time of his death, in 1909, the average number of reactors had been reduced from 22 to 5 per cent.

In administering this control program Dr. Pearson met with strong opposition, not only from ultra-conservative farmers but also from some highly reputable veterinarians. He was constantly involved in controversy and litigation. He traveled a rocky road with marked fortitude and forbearance. While serving as State Veterinarian he continued his duties at the School of Veterinary Medicine. Under the strain of over-work his health failed and he passed away while still in the prime of life, at the age of forty-one.

On December 16, 1936 the dairymen, health officers and veterinarians of Pennsylvania celebrated "Achievement Day." On that day the entire state was declared by the federal government to be a "modified accredited area," having less than half of one per cent of reactors. Truly this was a noteworthy achievement. It may be that there was an unseen guest at that banquet table, with a smile of quiet satisfaction at this triumphant consummation of his dreams.

JOSEPH TRIMBLE ROTHROCK, 1839–1922

Another Philadelphian to whom destiny beckoned and who followed her to glory was Joseph Trimble Rothrock. This Mifflin County youth studied Botany at Harvard University under the great Asa Gray, after which he graduated from the medical school of the University of Pennsylvania. He practiced medicine for a few years but his absorbing interest was plant life. In 1877, after participating in several governmental expeditions to explore the far west, he was appointed Professor of Botany at the University of Pennsylvania. He was the moving spirit in organizing the Pennsylvania Forestry Association, in 1886.

Forestry in Pennsylvania at that time was in a desperate condition. After 1840 lumbering was mostly by corporations having large financial resources and controlling very large tracts of timber. Clean cutting prevailed; few if any seed trees were left to renew the forest. Successive forest fires followed the lumbermen, destroying not only the new growth but also much of the soil cover. Quite frequently these fires were set deliberately by hunters and berry pickers. By 1900 the "Great Pennsylvania Desert" comprised more than 5,000,000 acres of cut-over and fire-devastated mountain land, mute witness to the greed and carelessness of man.

Eventually the warnings of Dr. Rothrock were heeded. In 1893 he was appointed to a commission to report on forestry conditions and needs in Pennsylvania. The report of this commission, which was prepared almost wholly by Dr. Rothrock, was the first comprehensive forestry program for this state. It was so convincing that in 1895 the General Assembly established a Division of Forestry in the state government and he was appointed the first Forestry Commissioner. For more than a quarter century thereafter he was an untiring and inspiring evangelist for forest conservation. He met and overcame bitter opposition from some private land owners who thought that his program would jeopardize their pecuniary interests.

The first objective of Dr. Rothrock was to prevent or check forest fires; it was futile to attempt reforestation until this was assured. He persuaded the Legislature to centralize responsibility for fire protection in state-employed fire wardens. He argued, "Taxes are paid for protection to property as well as persons. There is no sound reason why forest property should receive less protection than any other kind of

real estate." By 1905 the fire hazard had been greatly reduced and the reforestation of cut-over land through natural regeneration had begun.

The next step was forest tree planting, mainly on submarginal farm land. The first forest tree nursery was established at Mont Alto, Franklin County in 1902. From this and from other nurseries established subsequently, millions of seedlings have been supplied to citizens without charge, chiefly for planting on waste or eroded land.

The crowning achievement of Dr. Rothrock was the creation of state forests. In 1888, long before he entered public life, he had stated the principle involved: "Forests are useful for much more than the production of timber and firewood. They have a marked influence on the even flow of water in streams and springs and are even more useful for public recreation. The state should resume control over land that is too poor for farming and rough, mountain land from which the timber has been removed, especially at the headwaters of streams."

Acting on this principle, in 1898 the state authorized the purchase of large blocks of mountain land, most of which was tax-delinquent, as State Forest Reserves. During his administration half a million acres were acquired, at an average cost of $1.90 an acre. State Forests now comprise more than a million and a half acres. The investment is paying high dividends in water conservation and public recreation; eventually it will prove to have been a sound investment for timber production, also.

At the death of Dr. Rothrock, in 1922, the editor of Pennsylvania's leading farm journal paid him this tribute: "Pennsylvania state forests now cover a million and a quarter acres and every citizen is proud of them. That the state has these forests is due chiefly to the vision, courage and perseverance of Dr. Joseph Trimble Rothrock. He will not lack a memorial in Pennsylvania as long as trees grow for the pleasure and use of mankind."

The pruning hook of time has long since cut down these Philadelphia leaders of Pennsylvania agriculture, but this has only deepened our sense of obligation to them. The half million men and women on Pennsylvania farms join with us in paying tribute to their memory and to their devoted service.

THE RADIATION GREEN'S FUNCTIONS

BY

J. G. LINHART [1]

SYNOPSIS

This paper presents a method of calculating the radiation field of a group of charges by using the "radiation Green's function." The latter represents a concept similar to that of the Green's function in the potential theory, only its application is directed to the time variables problems of the radiation theory. Its usefulness will be demonstrated by examples taken from the problems connected with microwave oscillators.

1. INTRODUCTION

Most of the systems in nature are usually subjected to a number of complicated stimuli and only very rarely a case occurs in which the stimulus is of a simple mathematical character, resulting in a more or less simple response. In order to make the analysis of the general stimulus-response phenomena more accessible to mathematical treatment one often resolves the stimulus into a number of component stimuli. This is particularly effective in the analysis of systems that are governed by a law which allows the response to every individual stimulus to be considered separately from processes related to other stimuli. In such a case one may apply the superposition principle and obtain the total response as a simple sum (or an integral) of all the individual responses to all the component stimuli which go to represent the total stimulus.

The choice of the type of the component (or elementary) stimulus depends on the system, on its function and very often on convention, that is to say on the type of stimulus that was at one time considered fundamental in relation to the system.

However, three forms of elementary stimuli are used more often than any other type. These are the sin x (or cos x) function, the Dirac impulse function $\delta(x - x_0)$ and the Heaviside step function $H(x - x_0)$.

In the treatment of potential problems (for example, in electrostatics) the situation may be described by the Poisson's equation

$$\Delta\phi(x_1, x_2, x_3) = -4\pi\rho(x_1, x_2, x_3) \tag{1}$$

with ϕ satisfying certain boundary conditions. The $\rho(x_1, x_2, x_3)$ is the stimulus (in electrostatics the space charge density), the ϕ (the potential) is the response of the system to the presence of ρ (the system in electrostatics being the system of conductors and dielectrics).

[1] Research Laboratory of The British Thomson-Houston Co., Ltd., Rugby, England.

This equation may be difficult to solve, if $\rho(x_1, x_2, x_3)$ is a complicated function of x_1, x_2, x_3. In such a case it is easier to solve the equation

$$\Delta\phi(x_1, x_2, x_3) = -4\pi\rho(x_1', x_2', x_3')\,\delta(r - r') \qquad (1a)$$

where $\delta(r - r')$ is a three-dimensional Dirac function[2] and $r = \bar{a}_1 x_1 + \bar{a}_2 x_2 + \bar{a}_3 x_3$, where again the $\bar{a}_1$, $\bar{a}_2$, $\bar{a}_3$ are the unit vectors in the coordinate system x_1, x_2, x_3.

The solution of Eq. 1*a* represents the potential ϕ_0 at x_1, x_2, x_3 (the response) of a point charge ρ (component stimulus) at x_1', x_2', x_3'. Thus ϕ_0 will involve the x_1', x_2', x_3' as parameters and one may generally write

$$\phi_0 = \phi_0(x_1, x_2, x_3; x_1', x_2', x_3').$$

If $\rho(x_1', x_2', x_3') = 1$, the corresponding ϕ is the Green's function G of the system. Thus

$$\phi_0 = G(x_1, x_2, x_3; x_1', x_2', x_3')\,\rho(x_1', x_2', x_3').$$

Subsequently, once the $G(x_1, x_2, x_3; x_1', x_2', x_3')$ is found for a given system (that is, a given boundary) the total response $\phi(x_1, x_2, x_3)$ particular to a given $\rho(x_1', x_2', x_3')$ may be found by simple addition of the ϕ_0's. It is

$$\phi(x_1, x_2, x_3) = \int_{V'} \rho(x_1', x_2', x_3')\,G(x_1, x_2, x_3; x_1', x_2', x_3')\,dV' \qquad (1c)$$

where $dV' = g(x_1', x_2', x_3')dx_1'dx_2'dx_3'$ is the differential element of volume V' and $V' = V$ is the total volume of the system. The $\rho(x_1', x_2', x_3')$ acts as a weight function and $g(x_1', x_2', x_3')$ is a metrical function depending on the type of the coordinate system x_1, x_2, x_3 only.

A general solution Φ_s of Eq. 1 is then the sum of $\phi(x_1, x_2, x_3)$, which is the particular solution of Eq. 1 and of $\phi_1(x_1, x_2, x_3)$, which is a solution of the homogeneous equation

$$\Delta\phi = 0.$$

Thus

$$\Phi_s = \phi + \phi_1.$$

A similar method may be elaborated to solve the problems of electromagnetic field theory where a more general equation than Eq. 1 has to be solved, namely [3]

$$\Box\,\Psi(x_1, x_2, x_3, x_4) = -\frac{4\pi}{c}\,\Gamma(x_1, x_2, x_3, x_4) \qquad (2)$$

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[2] Normalized by $\int_v \delta(\bar{r} - \bar{r}')dv = 1$ (v = volume or the accessible space.)

[3] $\Box = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + \frac{\partial^2}{\partial w^2}$ in cartesian coordinates.

where the Ψ is the world potential, the Γ represents the four-current density and the $x_4 = ict$. This is a vector equation and in fact it is equivalent to four scalar equations. The Γ may also be written as

$$\begin{aligned}\Gamma &= \bar{a}_1\Gamma_1 + \bar{a}_2\Gamma_2 + \bar{a}_3\Gamma_3 + \bar{a}_4\Gamma_4 \\ &= q(\bar{a}_1 v_1 + \bar{a}_2 v_2 + \bar{a}_3 v_3 + \bar{a}_4 c).\end{aligned}$$

Equation 2 is generally difficult to solve, especially when $\Gamma(x_1, x_2, x_3, x_4)$ is a complicated function of x_1, x_2, x_3, x_4. We may again try to find a suitable Green's function here. This may be accomplished by finding the solutions of four vector equations:

$$\Box \Psi_1 = -\bar{a}_1 \delta(R - R') \tag{2.1}$$

$$\Box \Psi_2 = -\bar{a}_2 \delta(R - R') \tag{2.2}$$

$$\Box \Psi_3 = -\bar{a}_3 \delta(R - R') \tag{2.3}$$

$$\Box \Psi_4 = -\bar{a}_4 \delta(R - R') \quad (R = \text{world vector}). \tag{2.4}$$

Each of these represents actually four scalar equations.[4] For example, Eq. 2.1 is explicitly

$$\begin{aligned}[\Box \Psi_1]_{x_1} &= -\delta(R - R') \\ [\Box \Psi_1]_{x_2} &= [\Box \Psi_1]_{x_2} = [\Box \Psi_1]_{x_3} = 0\end{aligned}$$

where $[\ \ ]_{x_2}$ is a symbol selecting the x_i-th component of the expression in the bracket. The solutions of Eqs. 2.1, 2.2, 2.3, 2.4 will be the four radiation Green's functions of the system, that is

$$G_K = G_K(x_1, x_2, x_3, x_4; x_1', x_2', x_3', x_4') = G_K(R, R')$$

where $k = 1, 2, 3, 4$. G_K itself is of course a four vector, so that

$$G_k = \bar{a}_1 G_{k1} + \bar{a}_2 G_{k2} + \bar{a}_3 G_{k3} + \bar{a}_4 G_{k4}.$$

The meaning of G_k is the following: *G_k is the world potential at the point x_1, x_2, x_3 and at the time $x_4/ic = t$ generated by a unit point charge at the point x_1', x_2', x_3' possessing a unit velocity $v_k = c$ (in the a_k direction) at the time $t' = x_4'/ic$ and being stationary at all other times.* Knowing all 16 of the G_{ki} $(k, i = 1, 2, 3, 4)$, it is now possible to write down the solution of Eq. 2 (corresponds to Eq. 1*c*)

$$\Psi = \sum_{k=1}^{4} \int_{\Omega'} \Gamma_k(x, ', x_2', x_3', x_4') G_k(x_1, x_2, x_3, x_4; x_1', x_2', x_3', x_4')\, d\Omega'$$

$$\Psi = \sum_{i=1}^{4} \sum_{k=1}^{4} \int_{\Omega'} \bar{a}_i\, G_{ki}(R, R')\, \Gamma_k(R')\, d\Omega'. \tag{3}$$

[4] In cartesian coordinates only.

The principal difficulty of this treatment is the solution of Eqs. 2.1, 2.2, 2.3, 2.4. This difficulty may be obviated by considering any Ψ within the given boundaries to be composed of more fundamental fields, the so called eigen-fields of the system. The Ψ's will thus be expanded as a series of these component eigen-fields, which as we shall see presently simplifies Eqs. 2.1, etc., considerably.

2. THE EIGEN FUNCTION EXPANSION OF ELECTROMAGNETIC FIELD

The expansion of any electromagnetic field in terms of some fundamental set of fields may be accomplished in a manner similar to the expansion of a function of real variable as a Fourier series or Fourier-Bessel series. The conditions any set of elements in terms of which the field is expanded must satisfy are:

(*a*) The set must be complete.
(*b*) The members of the set must be mutually orthogonal.
(*c*) The set must be normalized.

There may be more than one set that may be used, but in problems of radiation theory one chooses the set formed by all the free electromagnetic vibrations (the modes) particular to the boundaries of the system (for example, of a cavity). Such a set is always infinite but in case of cavities of finite volume it is enumerable. Each member of the set represents a field pattern, capable of free vibrations at a definite frequency f_k. The resolution of a field into the component mode-fields procures a description of electromagnetic processes of great simplicity and elegance as will be seen presently.

The mathematical formulation of the new model has been developed by Rayleigh, Dirac, Fermi (1),[5] Weyl (2) and others. A very concise and clear exposition is given by Heitler (3). A part of the theory has been applied to the theory of resonant cavities by Schwinger and Casimir.

The Derivation of Basic Equations

The Maxwell's equations in the potential form may be written (in the Gaussian cgm. system):

$$\Delta \bar{A} - \frac{1}{c^2} \ddot{\bar{A}} = - \frac{4\pi}{c} \sum_k e_r \bar{v}_r \tag{4.1}$$

$$\Delta \Phi - \frac{1}{c^2} \ddot{\Phi} = - 4\pi \sum_k e_r \tag{4.2}$$

where e_r are the point charges, $\bar{v}_r$ their velocity vectors.

[5] The boldface numbers in parentheses refer to the references appended to this paper.

The Lorentz condition is:

$$\operatorname{div} \bar{A} + \frac{1}{c}\Phi = 0. \tag{4.3}$$

Let us distinguish the rotational and irrotational part of $\bar{A}$ and denote it $\bar{A}_1$ and $\bar{A}_2$, respectively. Thus:

$$\left.\begin{array}{l}\operatorname{div} \bar{A}_1 = 0 \\ \operatorname{curl} \bar{A}_2 = 0\end{array}\right\} \bar{A} = \bar{A}_1 + \bar{A}_2. \tag{4.4}$$

Thus

$$\bar{A}_2 = \text{const. grad. } P. \tag{4.5}$$

Now let us express $\bar{A}_1$, $\bar{A}_2$ and ϕ as series of mutually orthogonal components:

$$\bar{A}_1 = \sum_{\lambda} q_\lambda(t)\, \bar{A}_\lambda(x_1, x_2, x_3) \tag{5.1}$$

$$\bar{A}_2 = \sum_{\sigma} q_\sigma(t)\, \bar{A}_\sigma(x_1, x_2, x_3) \tag{5.2}$$

$$\Phi = \sum_{\mu} a_\mu(t)\, \Phi_\mu(x_1, x_2, x_3) \tag{5.3}$$

where the spatial part of each component satisfies the Helmholtz equations

$$\Delta \bar{A}_\lambda + \frac{\nu_\lambda^2}{c^2} \bar{A}_\lambda = 0 \tag{6.1}$$

$$\Delta \bar{A}_\sigma + \frac{\nu_\sigma^2}{c^2} \bar{A}_\sigma = 0 \tag{6.2}$$

$$\Delta \Phi_\mu + \frac{\nu_\mu^2}{c^2} \Phi_\mu = 0 \tag{6.3}$$

where ν_λ, ν_σ, and ν_μ are the eigen values of the system. It can be shown now that A_λ, A_σ are all mutually orthogonal within the given boundaries. Also all the ϕ_μ's are mutually orthogonal within the same boundaries and moreover the eigen values $\nu_\sigma \equiv \nu_\mu$ (*viz.*, ref. 3, p. 48). But in that case Eqs. 6.2 and 6.3 are identical since 6.2 may be written (using 4.5)

$$\Delta \operatorname{grad} P_\sigma + \frac{\nu_\sigma^2}{c^2} \operatorname{grad} P_\sigma = 0.$$

Therefore

$$\Delta P_\sigma + \frac{\nu_\sigma^2}{c^2} P_\sigma = 0.$$

Since P and ϕ satisfy the same boundary conditions, it follows that

$$P_\sigma = \Phi_\mu.$$

After substitution of the series for A_1, A_2, Φ into Eqs. 4.1 and 4.2, we get

$$\sum_\lambda \left(q_\lambda \Delta \bar{A}_\lambda - \frac{\ddot{q}}{c^2} \bar{A}_\lambda \right) + \sum_\sigma \left(q_\sigma \Delta \bar{A}_\sigma - \frac{\ddot{q}}{c^2} \bar{A}_\sigma \right) = - \frac{4\pi}{c} \sum_k e_r \bar{v}_r$$

$$\sum_\sigma \left(a_\sigma \Delta \Phi_\sigma - \frac{\ddot{a}_\sigma}{c^2} \Phi_\sigma \right) = - 4\pi \sum_k e_r.$$

Using Eqs. 6.1, 6.2, 6.3, one obtains

$$\sum_\lambda \frac{\bar{A}_\lambda}{c^2} (\ddot{q}_\lambda + \nu_\lambda^2 q_\lambda) + \sum_\sigma \frac{\bar{A}_\sigma}{c^2} (\ddot{q}_\sigma + \nu_\sigma^2 q_\sigma) = - \frac{4\pi}{c} \sum e_r \bar{v}_r \tag{7.1}$$

$$\sum_\sigma \frac{\Phi_\sigma}{c^2} (\ddot{a}_\sigma + \nu_\sigma^2 a_\sigma) = - 4\pi \sum_k e_r. \tag{7.2}$$

Multiplying Eq. 7.1 by $\bar{A}_\lambda$ and integrating over the cavity volume Ω we shall get

$$\ddot{q}_\lambda + \nu_\lambda^2 q_\lambda = \frac{1}{c} \sum_k e_r \bar{v}_r \bar{A}_\lambda(x_{1r}, x_{2r}, x_{3r}). \tag{7.3}$$

Multiplying Eq. 7.1 by $\bar{A}_\sigma$ and integrating over the cavity volume Ω we shall get

$$\ddot{q}_\sigma + \nu_\sigma^2 q_\sigma = \frac{1}{c} \sum_k e_r \bar{v}_r \bar{A}_\sigma(x_{1r}, x_{2r}, x_{3r}), \tag{7.4}$$

and similarly multiplying Eq. 7.2 by ϕ_σ and integrating over Ω we get

$$\ddot{a}_\sigma + \nu_\sigma^2 a_\sigma = \sum_k e_r \Phi_\sigma(x_{1r}, x_{2r}, x_{3r}) \tag{7.5}$$

where x_{1r}, x_{2r}, x_{3r} are coordinates of the rth charge e_r; also from the Lorentz equation we get a relation between $q_\sigma(t)$ and $a_\sigma(t)$

$$\nu_\sigma q_\sigma = a_\sigma$$

which, if satisfied for $t = t_0$ will always be satisfied. As q_λ, q_σ, a_σ represent the amplitudes of the eigen-fields one must normalize the A_λ, A_σ, ϕ_σ for the given space. That is effected by the convention

$$\int_\Omega \Phi_\sigma^2 d\Omega = \int_\Omega \bar{A}_\sigma^2 d\Omega = \int_\Omega \bar{A}_\lambda^2 d\Omega = 4\pi c^2. \tag{7.6}$$

3. THE INTEGRATION OF THE EXCITATION EQUATIONS BY MEANS OF THE RADIATION GREEN'S FUNCTION

The radiation part of the electromagnetic field is completely described by the solutions of Eq. 7.3. These again may be difficult to solve if the motion of the charges is complicated. Here however one may use the original idea of resolving the stimulus (which here is the motion of the charges) by means of Dirac impulse functions. Thus instead of solving Eq. 7.3 directly, we shall solve the following set of equations:

$$\ddot{q}_\lambda + \nu_\lambda^2 q_\lambda = \frac{1}{c} e\bar{v}\, \bar{A}_\lambda(x_1', x_2', x_3')\, \delta(t - t') \tag{8.7}$$

which represents the excitation q_λ of the eigen-field A_λ, by a charge e situated at x_1, x_2, x_3 and whose velocity is $\bar{v}\,\delta(t - t')$.

Instead of considering any vector $\bar{v}$, we may rewrite Eq. 8.1 for every component of $\bar{v}$ separately and moreover we may put

$$\bar{v} = \bar{a}_1 v_1 + \bar{a}_2 \bar{v}_2 + \bar{a}_3 v_3$$

where

$$v_1 = v_2 = v_3 = c, \quad e = 1.$$

Thus we get:

$$\ddot{\gamma}_{\lambda 1} + \nu_\lambda^2 \gamma_{\lambda 1} = A_{\lambda 1}(x_1', x_2', x_3')\, \delta(t - t') \tag{9.1}$$

$$\ddot{\gamma}_{\lambda 2} + \nu_\lambda^2 \gamma_{\lambda 2} = A_{\lambda 2}(x_1', x_2', x_3')\, \delta(t - t') \tag{9.2}$$

$$\gamma_{\lambda 3} + \nu_\lambda^2 \gamma_{\lambda 3} = A_{\lambda 3}(x_1', x_2', x_3')\, \delta(t - t') \tag{9.3}$$

where

$$\bar{A}_\lambda = \bar{a}_1 A_{\lambda 1} + \bar{a}_2 A_{\lambda 2} + \bar{a}_3 A_{\lambda 3}.$$

But this has brought us back to the concept of the radiation Green's function, only here we shall construct a restricted radiation Green's function, that is, a function related to the $\bar{A}_1$ part of the total electromagnetic field Ψ only. First of all we may obtain the solutions of Eq. 8.1 by a simple integration of the solutions of Eqs. 9.1, 9.2 and 9.3:

$$q_{\lambda i}(t) = \frac{1}{c} \sum_{i=1}^{3} \int_{-\infty}^{t} e(t')\, v_i(t').$$

$$\gamma_{\lambda i}[x_1'(t'), x_2'(t'), x_3'(t'), t]dt', \quad i = 1, 2, 3.$$

And for a group of N charges one obtains

$$q_\lambda(t) = \frac{1}{c} \sum_{n=1}^{N} \sum_{i=1}^{3} \int_{-\infty}^{t} e_n(t')\, v_{ni}(t')\gamma_{\lambda i}(t,t')dt'. \tag{10}$$

The radiation Green's function on the other hand will be the sum of all the $\gamma_{\lambda i}\, \bar{A}_\lambda$:

$$\bar{G}_i(t, t') = \sum_\lambda \gamma_{\lambda i}(t, t'; x_1', x_2', x_3')\, \bar{A}_\lambda(x_1, x_2, x_3) \quad i = 1, 2, 3. \tag{11}$$

Thus in fact we have three radiation Green's functions, each being a proper vector function. The complete radiation field $\bar{A}_1$ may then be obtained in the same way as the Ψ was obtained in Eq. 3.

$$\bar{A}_1 = \sum_{n=1}^{N} \sum_{i=1}^{3} \int_{\infty}^{t} e_n(t') v_{in}(t') \bar{G}_i(t, t') dt' = \sum_{\lambda} q_\lambda(t)\ \bar{A}_\lambda(x_1, x_2, x_3) \quad (12)$$

where N is the n^0 of charges in the cavity. For a continuous distribution of charge we get

$$\bar{A}_1 = \sum_{i=1}^{3} \int_{\Omega'} \int_{\infty}^{t} \rho(x_1', x_2', x_3', t') v_i(x_1', x_2', x_3', t') \times \bar{G}_1(x_1, x_2, x_3; x_1', x_2', x_3'; t, t') d\Omega' dt'. \quad (12.1)$$

The restricted radiation Green's function as given by Eq. 11 is still a too general concept, in fact the $\gamma_{\lambda i}$'s are far more useful as they furnish the means for the computation of the $q_\lambda(t)$'s. We shall call the $\gamma_{\lambda i}$'s the *mode-Green's functions* and we shall give their form for a cylindrical cavity in example No. 1. These are then the elementary response functions that we have set ourselves to discover.

So far we have been considering a non-lossy cavity, that is, the Q factor of such a cavity $Q = \infty$. It is possible, however, to use the mode-Green's functions even in a case of finite Q's, down to about $Q \doteq 10$. The approximation involved in such an extension of the eigen function method is related to the eigen functions being only approximately orthogonal and the volume integration indicated in Eq. 7.1 will leave a remainder containing terms such as

$$\int_{\Omega} \sum_{m} \bar{A}_m \bar{A}_n f_m(t)\, d\Omega = \sum_{m} f_m(t) \int_{\Omega} \bar{A}_m \bar{A}_n d\Omega \quad (\text{where } m \neq n).$$

These are the coupling terms between the various modes of the lossy cavity. It can be shown that these terms are of the order of $1/Q^2$ and thus provided $Q > 10$, we may still treat the mode patterns $\bar{A}_m$ and $\bar{A}_n$ as orthogonal.

Equations 7.3 represent a set of undamped oscillators, being acted on by forces described by the right hand side of the said equations. In order to take account of the damping (finite Q) let us proceed as follows.

By definition:

$$Q_\lambda = -\nu_\lambda \frac{W_\lambda}{\dot{W}_\lambda}.$$

Therefore

$$\ln \frac{W_\lambda}{W_{\lambda 0}} = -\frac{\nu_\lambda}{Q_\lambda} \quad \text{and} \quad W_\lambda = W_{\lambda 0}\, e^{-\frac{\nu_\lambda}{Q} t}$$

where $W_{\lambda 0}$ is the initial value of the stored energy.

The W_λ may be also expressed by the corresponding $q_\lambda(t)$ as follows:

$$W_\lambda = \frac{1}{8\pi c^2}\int_\Omega \{(\dot{q}A_\lambda)^2 + (cq \operatorname{curl} A_\lambda)^2\}d\Omega$$

and since for quasi-harmonic vibrations of frequency $\nu_\lambda/2\pi$ it holds:

$$\nu_\lambda A_\lambda = c \operatorname{curl} A_\lambda$$

we have

$$W_\lambda = \tfrac{1}{2}\{(\dot{q})^2 + \nu_\lambda^2 q_\lambda^2\}.$$

Also, as $\dot{q}_\lambda \cong \nu_\lambda q_\lambda$, it follows that

$$W_\lambda \cong \nu_\lambda^2 q_\lambda^2$$

and consequently

$$q_\lambda = q_{\lambda 0}\, e^{-\frac{1}{2}\frac{\nu}{Q}t}.$$

The corresponding differential equation is

$$\ddot{q}_\lambda + \frac{\nu_\lambda}{Q_\lambda}\dot{q}_\lambda + \nu_\lambda^2 q_\lambda = 0 \tag{13}$$

which is also the left hand side of Eq. 7.3 for a lossy cavity.

4. EXAMPLES

(a) A Point Charge Rotating in a Cylindrical Cavity (4)

The eigen-fields of a cylindrical cavity form two groups. The first group consists of the E-modes, that is, those modes that have an E_z (axial) component of the electric field but no H_z component. The second group is that of the H-modes, which possess an H_z but no E_z,

The E-field

$$\begin{aligned}
A_{\lambda z} &= a_0 J_n\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, r\right)\begin{matrix}\sin\\ \cos\end{matrix} n\theta \begin{matrix}\sin\\ \cos\end{matrix} xz \\
A_{\lambda\theta} &= a_0 J_n\left(\sqrt{\frac{\nu_\lambda^2}{c} - x^2}\, r\right)\frac{xn}{\left(\frac{\nu_\lambda^2}{c^2} - x^2\right)r}\begin{matrix}-\cos\\ \sin\end{matrix} n\theta \begin{matrix}\cos\\ -\sin\end{matrix} xz \qquad (14a)\\
A_{\lambda r} &= -\,a_0 J_n'\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, r\right)\frac{x}{\left(\frac{\nu_\lambda^2}{c^2} - x^2\right)^{\frac{1}{2}}}\begin{matrix}\sin\\ \cos\end{matrix} n\theta \begin{matrix}\cos\\ -\sin\end{matrix} xz.
\end{aligned}$$

The H-field:

$$A_{\lambda z} = 0$$

$$A_{\lambda\theta} = \frac{a_1}{x} J_n'\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, r\right) \begin{matrix}\sin \\ -\cos\end{matrix} n\theta \begin{matrix}\sin \\ \cos\end{matrix} xz \tag{14b}$$

$$A_{\lambda r} = \frac{a_1}{x} \frac{n}{\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}} \frac{J_n\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, r\right)}{r} \begin{matrix}\cos \\ \sin\end{matrix} n\theta \begin{matrix}\sin \\ \cos\end{matrix} xz \quad (xl = m\pi).$$

The coefficients a_0 and a_1 may be determined from the normalizing condition

$$\int_\Omega A_\lambda^2 d\Omega = 4\pi c^2$$

and using the boundary conditions at $r = R$ which for the E-modes is

$$J_n\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, R\right) = 0.$$

For the H-modes

$$J_n'\left(\sqrt{\frac{\nu_\lambda^2}{c^2} - x^2}\, R\right) = 0,$$

one obtains

$$a_0^2 = \frac{8c^4 s^2}{\pi l \nu_\lambda^2 R^2 J^2_{n-1}(sR)}$$

$$\left(\frac{a_1}{x}\right)^2 = \frac{8c^2 s^2}{\pi l[(s^2R^2 + 1) - (n+1)^2]J_n^2(sR)}$$

where l = length of the cavity and

$$s^2 = \left(\frac{\nu_\lambda}{c}\right)^2 - x^2.$$

Thus Eqs. 9 will become:

The E-field equations

$$\ddot{\gamma}_{\lambda z} + \nu_\lambda^2 \gamma_{\lambda z} = a_0 J_n(sr_0) \begin{matrix}\sin \\ \cos\end{matrix} n\theta_0 \begin{matrix}\sin \\ \cos\end{matrix} xz_0\, \delta(t - t')$$

$$\ddot{\gamma}_{\lambda\psi} + \nu_\lambda^2 \gamma_{\lambda\psi} = a_0 \frac{J_n(sr_0)xn}{s^2 r_0} \begin{matrix}-\cos \\ \sin\end{matrix} n\theta_0 \begin{matrix}\cos \\ -\sin\end{matrix} xz_0\, \delta(t - t') \tag{15a}$$

$$\ddot{\gamma}_{\lambda r} + \nu_\lambda^2 \gamma_{\lambda r} = -a_0 \frac{x}{s} J_n'(sr_0) \begin{matrix}\sin \\ \cos\end{matrix} n\theta_0 \begin{matrix}\cos \\ -\sin\end{matrix} xz_0\, \delta(t - t')$$

The H-field equations

$$\begin{aligned}\ddot{\gamma}_{\lambda\psi} + \nu_\lambda^2\gamma_{\lambda\psi} &= \frac{a_1}{x} J_n'(sr_0) \begin{matrix}\sin\\ -\cos\end{matrix} n\theta_0 \begin{matrix}\sin\\ \cos\end{matrix} xz_0\,\delta(t-t') \\ \ddot{\gamma}_{\lambda r} + \nu_\lambda^2\gamma_{\lambda r} &= \frac{a_1}{x}\frac{n}{s}\frac{J_n(sr_0)}{r_0} \begin{matrix}\cos\\ \sin\end{matrix} n\theta_0 \begin{matrix}\sin\\ \cos\end{matrix} xz_0\,\delta(t-t')\end{aligned} \tag{15b}$$

where r_0, θ_0, z_0 are the coordinates of the position of the charge.

The Laplace transforms of these equations are all of the form

$$\bar{\gamma}_{\lambda i}(p) = A_{\lambda i}\frac{p}{p^2+\nu_\lambda^2}e^{-t'p}$$

where $A_{\lambda i}$ is time-independent part of the right hand side of Eqs. 15a and 15b.

Using Duhamel's theorem we get

$$\gamma_{\lambda i}(t_1, t') = A_{\lambda i}\int_0^t \cos\nu_\lambda(t-\tau)\,H(\tau-t')\,d\tau.$$

Therefore,

$$\gamma_{\lambda i}(t_1, t') = \frac{A_{\lambda i}}{\nu_\lambda}\sin\nu_\lambda(t-t')\,H(t-t').$$

Having obtained these mode-Green's functions we may proceed to obtain the $q_\lambda(t)$'s for all modes. According to Eq. 10 one gets

$$q_\lambda(t) = \frac{e}{c}\int_{t'=-\infty}^{t'=t} v_\lambda(t')\,\gamma_{\lambda\psi}(t_1, t')dt'$$

since

$$v_r(t) = v_z(t) = 0 \quad \text{and} \quad e(t) = \text{const.}$$

Let us first investigate the case of uniform rotation which has started at $t' = 0$, that is, the case of $v_\psi(t') = r_0\omega_0 H(t-t')$. Thus it holds for the E-fields

$$\begin{aligned} q_\lambda(t) &= e\beta a_0\frac{J_n(sr_0)xn}{s^2r_0\nu_\lambda}\int_0^t \begin{matrix}-\cos\\ \sin\end{matrix} n\omega_0 t'\sin\nu_\lambda(t-t')\,H(t-t')dt' \qquad (16)\\ &= C\int_0^t \left\{ \begin{matrix}-\sin\\ \cos\end{matrix}[(n\omega_0+\nu_\lambda)t'+\nu_\lambda t] - \begin{matrix}\sin\\ \cos\end{matrix}[(n\omega_0-\nu_\lambda)t'+\nu_\lambda t]\right\}dt'\end{aligned}$$

where

$$C = \tfrac{1}{2}e\beta\frac{a_0x}{s^2r_0\nu_\lambda}n\,J_n(sr_0).$$

Integrating we get

$$q(t) = 2C\left\{\frac{\sin \frac{1}{2}(n\omega_0 + \nu_\lambda)t}{n\omega_0 + \nu_\lambda} \begin{matrix}\sin \\ -\cos\end{matrix} \tfrac{1}{2}(n\omega_0 - \nu_\lambda)t \right.$$
$$\left. - \frac{\sin \frac{1}{2}(n\omega_0 - \nu_\lambda)t}{n\omega_0 - \nu_\lambda} \begin{matrix}-\sin \\ \cos\end{matrix} \tfrac{1}{2}(n\omega_0 + \nu_\lambda)t \right\}. \quad (16a)$$

For $n\omega_0 = \pm \nu_\lambda$ we shall have

$$q_{n\omega_0} = \pm C\, t \begin{matrix}\sin \\ \cos\end{matrix} n\omega_0 t.$$

Thus if a synchronism between the angular velocity of rotation of the charge and that of one of the resonant fields is maintained, the excitation coordinate $q_\lambda(t)$ grows linearly with time.

The energy stored in the field of an individual mode is

$$W_\lambda = W_\lambda^{El} + W_\lambda^{Mag}.$$

The energy input $\dot{W}_\lambda$ into the λ field may be derived from the dynamical analogy of Eq. 7.3 with the equations of forced oscillations:

$$\dot{W}_\lambda = \frac{\partial W_\lambda}{\partial t} = \tfrac{1}{2}\{2\ddot{q}\dot{q} + 2\nu_\lambda^2 \dot{q}q\}$$
$$= \dot{q}(\ddot{q} + \nu_\lambda^2 q)$$
$$\dot{W}_\lambda = \dot{q}F$$

where F is the left hand side of Eq. 7.3. Thus in our case $\dot{W}$ for the E-field is

$$\dot{W}_{(n\omega_0)} = \tfrac{1}{4}e^2\beta^2\left[\frac{a_0 x n}{s^2 r_0} J_n(sr_0)\right]^2 t,$$

and for the H-field

$$\dot{W}_{(n\omega_0)} = \tfrac{1}{4}e^2\beta^2\left[\frac{a_1}{x} n\omega_0 J_n'(sr_0)\right]^2 t.$$

Let us now investigate a lossy cavity. Equation 9 will become (*viz.* Eq. 13)

$$\ddot{\gamma}_{\lambda i} + \frac{\nu_\lambda}{Q_\lambda}\dot{\gamma}_{\lambda i} + \nu_\lambda^2 \gamma_{\lambda i} = A_{\lambda i}\,\delta(t - t'); \quad i = r, z, \psi.$$

The Laplace transform of these equations is

$$\bar{\gamma}_{\lambda i}(p) = A_{\lambda i}\frac{p}{p^2 + \frac{\nu_\lambda}{Q_\lambda}p + \nu_\lambda^2} e^{-t'p}.$$

Using Duhamel's theorem we get

$$\gamma_{\lambda i}(t, t') = A_{\lambda i} \int_0^t \frac{-\nu_\lambda}{\sqrt{\nu_\lambda^2 \left(1 - \frac{1}{4Q_\lambda^2}\right)}} e^{-\frac{1}{2}\frac{\nu_\lambda}{Q}(t-\tau)} \times \sin\left\{\sqrt{\nu_\lambda^2 \left(1 - \frac{1}{4Q_\lambda^2}\right)}\,(t-\tau) - \phi\right\} H(\tau - t')d\tau.$$

For $Q_\lambda \gg 1$ it is

$$\gamma_{\lambda i}(t_1, t') = -A_{\lambda i} \int_{t'}^t e^{-\frac{1}{2}\frac{\nu_\lambda}{Q}(t-\tau)} \sin\left[\nu_\lambda(t-\tau) - \phi\right] d\tau$$

where

$$tg\phi = 2Q_\lambda \quad \text{and} \quad \phi \cong \frac{\pi}{2}$$

and

$$\gamma_{\lambda i}(t_1, t') \cong \frac{A_{\lambda i}}{\nu_\lambda} e^{-\frac{\nu_\lambda}{2Q_\lambda}(t-t')} \sin \nu_\lambda(t-t')\, H(t-t').$$

These are the mode-Green's functions for a lossy cavity.

The $q_\lambda(t)$ will be, similarly to Eq. 16,

$$q_\lambda(t) = 2C \int_0^t \frac{-\cos}{\sin} n\omega_0 t' \sin \nu_\lambda(t-t')\, e^{-\frac{\nu_\lambda}{2Q_\lambda}(t-t')} H(t-t')dt' \tag{17}$$

$$= 2C\, e^{-\frac{\nu_\lambda}{2Q_\lambda}t} \left\{ \sin \nu_\lambda t \int_0^t \frac{-\cos}{\sin} n\omega_0 t' \cos \nu_\lambda t'\, e^{+\frac{\nu_\lambda}{2Q_\lambda}t'} dt' - \cos \nu_\lambda t \int_0^t \frac{-\cos}{\sin} n\omega_0 t' \sin \nu_\lambda t'\, e^{\frac{\nu_\lambda}{2Q_\lambda}t'} dt' \right\}$$

Let us put (dropping the subscript λ)

$$\sin \psi = \frac{-2a\Omega}{\sqrt{(\Omega^2 - \nu^2)^2 + 2a^2(\Omega^2 + \nu^2)}}$$

$$\cos \psi = \frac{\nu^2 - \Omega^2 + a^2}{\sqrt{(\Omega^2 - \nu^2) + 2a^2(\Omega^2 + \nu^2)}},$$

where $a = \frac{\nu}{2Q}$. Thus one obtains

$$q \cong \frac{2\nu C}{\sqrt{(\Omega^2 - \nu^2) + \frac{1}{2Q^2}(\Omega^2 + \nu^2)\nu^2}} \frac{\cos}{-\sin}(n\omega_0 t - \psi) - 2\nu C\, e^{-\frac{\nu}{2Q}t} \frac{(\nu^2 - \Omega^2) + a^2}{(\Omega^2 - \nu^2)^2 + \frac{1}{2Q^2}(\Omega^2 + \nu^2)\nu^2}, \tag{18}$$

which corresponds to Eq. 16*a*.

5. CONCLUSION

The description of generation of electromagnetic radiation by moving charges by means of the mode-Green's functions appears to possess a simplicity not encountered in the continuum methods of classical theory of electromagnetic field. It presents a model of interaction between a unit quantum of movement of a unit point charge and the corresponding excitation of a set of eigen-fields. The formulation may itself have no other meaning than that of a convenient mathematical artifice, but on the other hand it may show its bearing on such problems as the action at distance and the continuity of movement and space.

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CALCULATION OF CAMERA EXPOSURE

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

1. INTRODUCTION

This paper is not written for the expert photographer. It is not written for Junior and the Girl Friend or for Uncle John on his yearly outing. It is for the scientifically trained man who has occasional use for photography as a scientific instrument. In such cases, a pressing question is that of *camera exposure.*

Most treatments of the subject are on an elementary level, with all mathematics carefully eliminated to meet the requirements of the average amateur. Obviously, such rule-of-thumb instructions are not what the scientist wants, and their use leads to unwarranted waste of time and film.

There is nothing radically new in this paper, but it does differ from previous formulations in the following respects:

(1) It presents a unified picture of ordinary and flash photography, using the mks. system of units throughout.
(2) It shows exactly what approximations are made.
(3) It indicates that "correct exposure time" is a figment of the imagination.

(1) One of the goals in the development of scientific nomenclature is *universality*—the breaking down of departmentalism so that all branches of science and engineering will associate the same symbol and the same unit with a given concept. Any other scheme entails unnecessary confusion and waste of time. The mks. system has achieved world-wide recognition and should be used in photography just as much as in electromagnetism or in mechanics.

Strangely enough, the accepted unit of photographic "exposure" is the mks. unit (lumen sec m^{-2}). But most equations contain a strange mixture of quantities, and measurements are made in all sorts of peculiar units. For instance, the Weston exposure meter reads in "candles per square foot" while the G.E. Lightmeter reads in "foot-candles." Despite the antiquarian similarity of the names, the units are completely different and neither has any excuse for its existence.

(2) Apparently the assumption has always been made that a method of predicting photographic exposure time is worthless unless it has been

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reduced to the kindergarten level. In such simplification, many factors must be neglected; but they can be neglected safely only if the user is cognizant of them. For example, even an experienced photographer sometimes forgets that, at short object distances, the *f*-numbers engraved on his lens have no direct correlation with exposure.

(3) Even the most sophisticated treatises seem to make the tacit assumption that for every scene there is a unique exposure time. This assumption is false. In general, the total helios range (from the darkest tiny detail in the scene to the brightest detail) is either

(*a*) Less than the range that can be accurately reproduced by the particular photographic process, or

(*b*) Greater than the range that can be reproduced.

If (*a*), then an infinite number of exposure times exist, any one of which allows satisfactory rendition. If (*b*), then no exposure time will allow correct rendition of the scene, and the photographer must decide which end of the scale he will sacrifice and how much. In neither case is there a unique exposure time which can be determined by taking one reading of any conceivable type of meter. And only if the photographer has the complete story—the helios distribution of the scene and the characteristic of the film—can he make an intelligent decision.

We advocate the *two-helios method* for the calculation of camera exposure: the measurement of two values of helios (H_{max} and H_{min}) and the decision as to the placement of these two points on the characteristic curve of the film. Any attempt to eliminate the decisions of the photographer and thus turn him into a robot is sure to produce inferior results.

2. PHOTOMETRICS

Photometric nomenclature is still in a chaotic state. Photography (**1**),[3] colorimetry (**2**), astronomy, and illuminating engineering (**3**) has each developed a specialized nomenclature having little relation to the needs of other branches of science. The system of Table I is believed to be the only notation developed to date that is of sufficient generality to cover all applications of light (**4**).

The basic quantity may be taken as *pharos F* or luminous flux, which is universally expressed in *lumens*. An associated concept is *pharosage D* or luminous flux density. Pharosage is really a vector quantity (**5**), though for our present purpose only the magnitude is needed. Analogous to *F* and *D* are the quantities containing time: *phos Q* and *phosage U*, the latter being particularly useful in photography. The only other quantity needed in this work is *helios*, a sort of generalized brightness. It may be measured by employing a photocell (which

[3] The boldface numbers in parentheses refer to the references appended to this paper.

TABLE I.—*Photometric Quantities.*

Symbol	Name	Dimensions	mks Unit
F	pharos (luminous flux)	$[F]$	lumen
D	pharosage (luminous flux density) $\left\{\begin{array}{l} D_i = \text{incident pharosage (illumination)}, \\ D_e = \text{reflected or transmitted pharosage} \end{array}\right.$ For reflecting surface, $D_e = \rho D_i$ For transmitting surface, $D_e = \tau D_i$	$[FL_i^{-2}]$	lumen m^{-2}
H	helios (brightness) $H = \pi \lim_{\Omega \to 0} (D_i/\Omega)$ For perfectly diffusing surfaces, $H = D_e$		blondel
Q	phos (quantity of light) $Q = \int F dt$	$[FT]$	lumen sec
U	phosage ("exposure") $U = \int D dt$	$[FTL_i^{-2}]$	lumen sec m^{-2}

where ρ = reflectance of a surface (numeric),
τ = transmittance (numeric),
Ω = solid angle (steradian),
t = time (sec.).

measures D_i) fitted with an opaque shield or a lens which limits the incoming light to a cone (4) with solid angle Ω. The instrument is pointed in any direction, and the ratio D_i/Ω is a measure of the helios in that direction.

We also need the equation for the pharosage from a luminous disk. The incident pharosage at any point Q on a surface that is parallel to the disk is given by the simple formula,

$$D_Q = H_Q \sin^2 \gamma, \tag{1}$$

where

H_Q = helios of disk (blondel) as seen from point Q,
γ = minimum plane angle from axis of convergent lightcone to edge of lightcone.

Equation 1 is exact, the only requirement being that H is the same for all rays in the lightcone (6). The equation can be written also in terms of the dimensions

$$D_Q = \frac{H_Q}{4p_1p_2}[4a^2 - (p_1 - p_2)^2], \tag{2}$$

where

a = radius of disk (m),
x = distance (m) from axis to point Q,
l_2 = distance (m) between disk and illuminated surface,

$$\left.\begin{aligned} p_1^2 &= (l_2^2 + x^2 + a^2) + 2ax, \\ p_2^2 &= (l_2^2 + x^2 + a^2) - 2ax. \end{aligned}\right\} \tag{3}$$

For the special case of point Q on the axis ($x = 0$), Eq. 2 reduces to

$$D_Q = H_Q a^2/(l_2^2 + a^2). \tag{2a}$$

We now derive an equation for the pharosage incident on the film, produced by the given helios distribution of the scene (Fig. 1). A

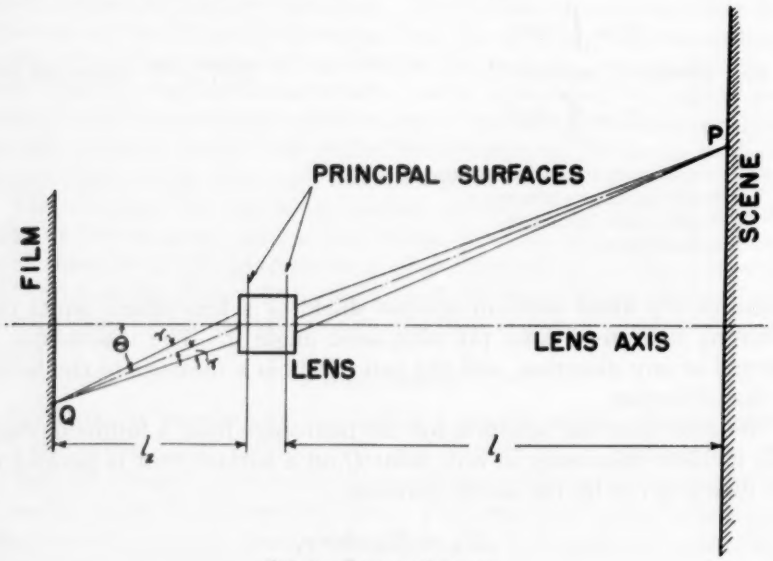

FIG. 1. Scene and camera.

plane in the object space is imaged on the film, distances to the principal surfaces of the lens being l_1 and l_2. The scene represents a specific helios distribution which ordinarily varies from point to point. Consider a definite point P, in which direction the helios is H_P as measured in front of the lens. The helios as measured from Q is almost the same (7) as H_P but is slightly reduced because of absorption and scattering in the lens:

$$H_Q = \tau H_P,$$

where τ is the transmittance of the lens at the angle Θ.

Neglecting possible vignetting, we find that the lens viewed from Q appears as a luminous circular disk of helios τH_P. Thus, from Eq. 2, the incident pharosage (lumen m^{-2}) received on the film at Q is

$$D_Q = \frac{\tau H_P}{4p_1p_2}[4a^2 - (p_1 - p_2)^2]. \tag{2b}$$

For any point P in the object space, there is a corresponding point Q in the image space, and the incident pharosage at the latter point is directly proportional to the helios of the particular part of the scene being imaged at Q.

From lens theory,

$$1/l_1 + 1/l_2 = 1/l_f \tag{3}$$

or

$$l_2 = \frac{l_1}{(l_1/l_f) - 1}, \tag{4}$$

where l_f is the focal length of the lens. It is convenient also to introduce the *relative aperture* (f-number):

$$f = l_f/2a. \tag{5}$$

Then Eq. 3 becomes

$$p^2 = l_f^2\left[\frac{(l_1/l_f)^2}{[(l_1/l_f) - 1]^2} + \frac{1}{4f^2} + (x/l_f)^2\right] \pm l_f x/f, \tag{6}$$

where the plus sign is employed for p_1 and the negative sign for p_2. The incident pharosage at any point on the film is given by Eq. 2*b* with p obtained from Eq. 6. The phosage (lumen sec m^{-2}) is, of course, obtained by multiplying the incident pharosage by the exposure time, or

$$U = \frac{\tau H_P t}{4p_1p_2}[(l_f/f)^2 - (p_1 - p_2)^2], \tag{7}$$

where t is the *effective exposure time* (sec) as marked on the shutter (8).

Equation 7 is an exact expression, applicable in any case, p_1 and p_2 being obtained from Eq. 6. In most applications, however, the distance from subject to camera is much greater than the focal length of the lens and $a \ll l_f$. Then $l_1 \gg l_f$, so Eq. 6 reduces to

$$p^2 \cong l_f^2[1 + (x/l_f)^2]\left\{1 \pm \frac{(x/l_f)}{f[1 + (x/l_f)^2]}\right\}$$

or

$$p \cong l_f[1 + (x/l_f)^2]^{\frac{1}{2}}\left\{1 \pm \frac{1}{2f}\frac{(x/l_f)}{1 + (x/l_f)^2}\right\}.$$

Then

$$p_1 - p_2 \cong (x/f)[1 + (x/l_f)^2]^{-1}$$

and

$$p_1 p_2 \cong l_f^2[1 + (x/l_f)^2].$$

Substitution into Eq. 7 gives

$$U = \frac{\tau H_p t}{4f^2[1 + (x/l_f)^2]^2},$$

and since $x/l_f = \tan \Theta$,

$$U = \frac{\tau H_p t}{4f^2} \cos^4 \Theta. \tag{8}$$

Thus the phosage on the film varies as $\cos^4 \Theta$ when Q moves from the lens axis. Vignetting may reduce phosage still further, as shown in Section 4.

In the remainder of the paper, Eq. 8 will be used as basic equation instead of Eq. 7. The photographer, however, should realize the approximations that are involved. In the first place, the distance from film to lens is taken as l_f, which is strictly true only when the lens is focussed on infinity. In photographing an object full size, for instance, $l_1 = l_2 = 2l_f$ and the use of Eq. 8 without correction will result in underexposure by a factor of 4. In the second place, we have assumed in Eq. 8 that $a \ll l_f$. At $f = 1$, this assumption introduces an error of 20 per cent. For most cases encountered in practice, the approximate expression, Eq. 8, is satisfactory; but in doubtful cases, use the exact expression, Eq. 7.

3. THE SCENE

To calculate camera exposure in a scientific manner, the photographer must have data on (*a*) the scene or subject to be photographed, (*b*) the lens, and (*c*) the film.

Under (*a*), he must know the helios distribution of the scene. These data are obtained preferably by use of a narrow-field helios meter set on a tripod near the camera. The instrument is pointed at various details in the field, and values of helios are measured. A visual instrument, the Luckiesh-Taylor Helios Meter (**9**), has been used for this purpose in a number of investigations (**10**). Narrow-field photoelectric instruments are also possible (**11**), though none is commercially available at present. An ideal exposure meter should have a field of one degree or less, should produce a minimum of scattered light, and should be adjustable to the spectral response of the film being used.

Lacking such an instrument, one may still obtain reasonably good results by placing an ordinary wide-angle photoelectric exposure meter (**12**) close to the scene detail being measured. Care must be taken not to cast a shadow on the detail or to reflect additional light onto it. If a

Weston exposure meter is employed, the readings are multiplied by 33.8 to obtain helios in blondels.

Ordinarily a complete helios survey is unnecessary: the photographer needs only the maximum and minimum values of helios *for the details in which gradation is desired.* The *total* range of the scene, from the brightest detail to the darkest shadow, is completely unimportant (13).

TABLE II.—*Typical Values of Helios.*

Outdoor, front-lighted scenes near sea level

Detail	Helios (blondels)	
Clear day		
Blue sky (near horizon)	12,000	
Clouds	35,000	
	In sunlight	In shadow
Face (maximum)	18,000	2,000
Grass	8,000	400
Trees (foliage)	3,000	400
Sidewalk	12,000	1,000
Macadam pavement	7,000	700
Buildings		
White	50,000	5,000
Granite	15,000	1,500
Yellow brick	12,000	1,200
Red brick	6,000	600
Dense shadows		50
Overcast sky (light)		
Sky (near horizon)	20,000	
Face	4,000	
Grass	1,000	
Trees	1,000	
Sidewalk	2,500	
Macadam pavement	1,500	
Buildings		
White	11,000	
Granite	3,500	
Yellow brick	3,000	
Red brick	1,500	
Dense shadows	100	

But we do need *two* definite readings, not *one.* The popular scheme, employing a photoelectric exposure meter at the camera to obtain an average over the scene, is worthless, as has been shown by test (14).

Take, for instance, an outdoor portrait in the shade. The average helios of the face is found to be 2000 blondel and the helios of the sky is 12,000 blondel. If the subject is posed in the shade with a sky background, the ordinary exposure meter (60° lightcone) at the camera will

TABLE III.—*Over-All Transmittance of Lenses.*

(Coated lenses, no absorption, no vignetting)

		$\tau \cdot \zeta(\Theta)$				
Θ	Pinhole ($n = 0$)	$n = 1$	2	3	4	5
0	1.000	0.970	0.940	0.912	0.883	0.856
5°	0.985	0.955	0.926	0.899	0.870	0.844
10	0.941	0.912	0.885	0.857	0.831	0.806
15	0.870	0.845	0.819	0.793	0.768	0.745
20	0.780	0.755	0.733	0.711	0.688	0.668
25	0.675	0.655	0.635	0.615	0.596	0.578
30	0.562	0.546	0.528	0.512	0.497	0.482
35	0.450	0.436	0.423	0.410	0.397	0.386
40	0.344	0.334	0.324	0.314	0.304	0.295
45	0.250	0.243	0.235	0.228	0.221	0.214

TABLE IV.—*Number of Air-Separated Lens Elements in a Few Camera Lenses.*

Manufacturer	Name of lens	l_f, mm.	f	n
E. K. Co.	Anastar	48	4.5	3
	Anaston	105	4.5	3
	Ektar	44	3.5	3
	Retina-Xenon	50	2	4
	Anastar	80	3.5	4
	Ektar	152	4.5	3
Leitz	Hektor	28	6.3	3
	Summaron	35	3.5	4
	Elmar	50	3.5	3
	Summitar	50	2	4
	Summicron	50	2	6
	Summarit	50	1.5	5
	Summarex	85	1.5	5
	Elmar	90	4	3
	Hektor	135	4.5	3
	Telyt	200	4.5	4
	Telyt	400	5.0	4
Zeiss	Protar		12.5	2
	Tessar		4.5	3
	Unar		6	4
	Sonnar		2	3
	Triotar		4	3

read approximately 10,000 blondel. The resulting exposure time, read from the meter, is 1/300 sec at $f = 2.8$ for Kodachrome. The same subject, posed in front of a mass of foliage ($H = 400$ blondel), with exactly the same helios for the face, should have identical exposure. But the meter reads 450 blondel and indicates an exposure time of 1/15 sec at $f = 2.8$. Here is a ratio of 20:1!

A single reading does not give enough information: *the necessary and sufficient data for the scene consist of H_{max} and H_{min} for the range over which detail is desired.* Table II gives a rough idea of helios values to be expected in outdoor scenes.

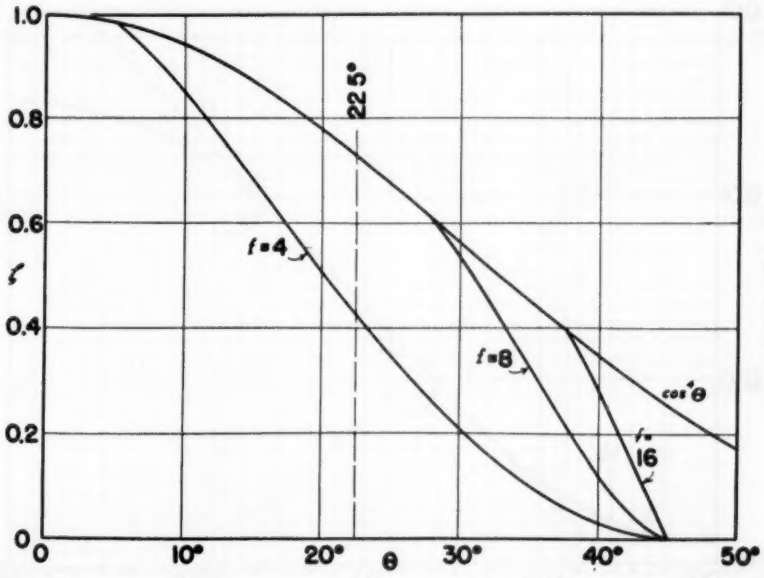

FIG. 2. Relative pharosage incident on the film, as a function of the angle Θ from the lens axis. The ordinates represent relative pharosage,

$$\zeta = D(\Theta)/D(0).$$

The upper curve applies to any lens having no vignetting. The other curves depend greatly on the details of lens and mount and are calculated for a specific lens with maximum $f = 4$. The vertical dotted line indicates the outer limit of the range utilized in an ordinary camera with $2\Theta = 45°$.

4. THE LENS

The lens affects camera exposure because of reflection and absorption of light. Also, at points not on the axis, the incident pharosage is reduced according to the $\cos^4$-relation and by possible vignetting. At each glass-air surface in the lens, some light is reflected. Measurement indicates a transmittance of 0.960 for each uncoated surface and 0.985 for each coated surface. Absorption can usually be neglected, particularly when using color film or a panchromatic film with green filter.

Thus a lens containing 5 elements separated by air spaces will have an approximate transmittance τ of $(0.960)^{10} = 0.66$ if uncoated or 0.85 if coated. Data are given in Tables III and IV.

These values are for points on the lens axis. At an angle Θ from the axis, the incident pharosage on the film is reduced by an additional multiplier,

$$\zeta = D(\Theta)/D(0).$$

With no vignetting, $\zeta = \cos^4 \Theta$. This factor is troublesome in photographic photometry (15), and even in ordinary photography with wide-angle lenses.

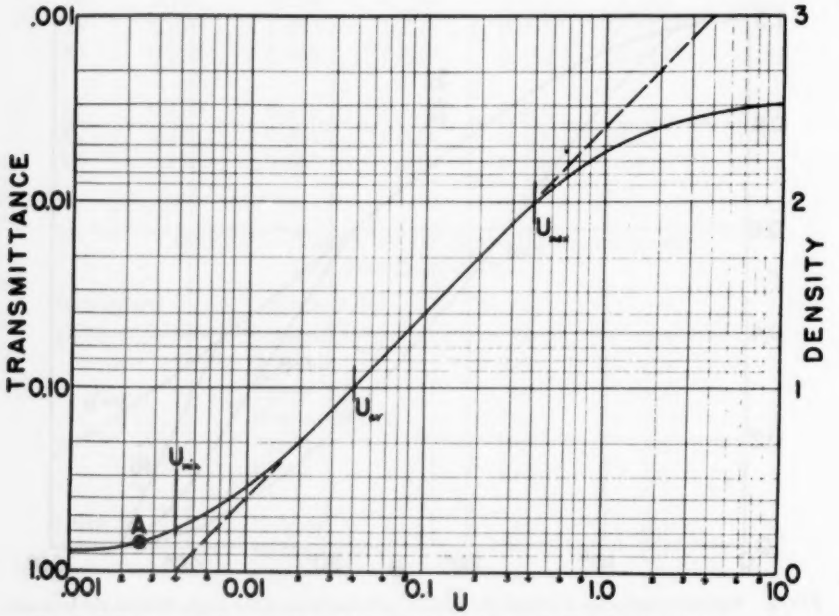

FIG. 3. Characteristic curve for a high-speed panchromatic film developed to $\gamma = 1.0$.

Ordinates are transmittance of the finished negative, abscissae are values of incident phosage (lumen sec. m^{-2}) on the film. The A.S.A. rating is based on point A, where the slope of the curve is 0.3 of the average slope over a range of 1.5 log units. U_{max} and U_{min} are suggested limits of phosage, U_{av} is the geometric mean:

$$U_{av} = [U_{max}U_{min}]^{\frac{1}{2}}.$$

At points off the axis, a further reduction in pharosage may be caused by vignetting (16). This factor depends on constructional details of the lens and mount, so no general results can be given. Figure 2 shows calculated vignetting for a certain lens with $f = 4$. The ordinates represent the factor ζ, including both $\cos^4 \Theta$ and vignetting. The top curve is for no vignetting. With the lens at full aperture, however, the mount begins to cut off the light at $\Theta = 5°$, and no light can get through at $\Theta = 45°$. With the lens stopped down, however, vignetting is much less troublesome. Note that the resulting curves of ζ have

discontinuous derivatives at the angles where the lens barrel begins to obstruct the light. von Gruber (17) and Washer (18) are in error in trying to represent their experimental data by smooth curves.

Table III indicates how the factor $\tau\zeta$ is affected by Θ and by the axial transmittance of the lens. The lenses are assumed to be coated, to have negligible absorption and to be stopped down sufficiently to eliminate vignetting. In calculating camera exposure, it is usually satisfactory to employ an average value, as will be done in the remainder of the paper. If $\tau = 0.90$, $\Theta = 15°$, and vignetting reduces the pharosage by 15 per cent, then $4/\tau\zeta = 6.0$ and Eq. 8 becomes

$$t/f^2 = 6U/H. \tag{9}$$

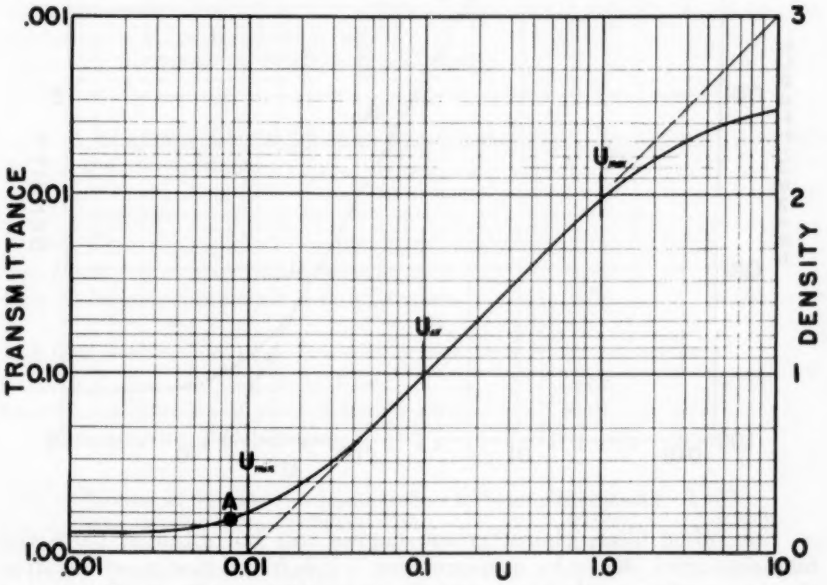

FIG. 4. Characteristic curve for a medium-speed panchromatic film (A.S.A. 32) developed to a gamma of unity.

5. THE FILM

A typical curve for a high-speed panchromatic negative (A.S.A. rating 100) is shown in Fig. 3. Ordinates represent transmittance τ of the developed negative; abscissae represent phosage U (lumen sec m^{-2}) incident on the film. Jones (19) has shown that excellent prints can be obtained when using the curve even as far down as point A. But results at these low values of phosage are greatly affected by scattered light in lens and camera, so it is not advisable to use much of the toe of the curve. Safe limits U_{min} and U_{max} are marked on the curve, more or less arbitrarily. The geometric mean of U_{min} and U_{max} is denoted by U_{av}. Figure 4 gives a curve for a medium-speed panchromatic film

(A.S.A. rating 32). The curve is essentially the same as that of Fig. 3, but it has been shifted along the U-axis. The ratio of U_{max} to U_{min} is 100 in either case.

Figure 5 shows a curve for a color transparency (Weston speed 8) obtained by reversal of the exposed color film. The ratio U_{max}/U_{min} is only 10 in this case, instead of 100 as obtained in Figs. 3 and 4. This means that exposure time for a reversal process must be predicted very accurately if a good transparency is to be produced.

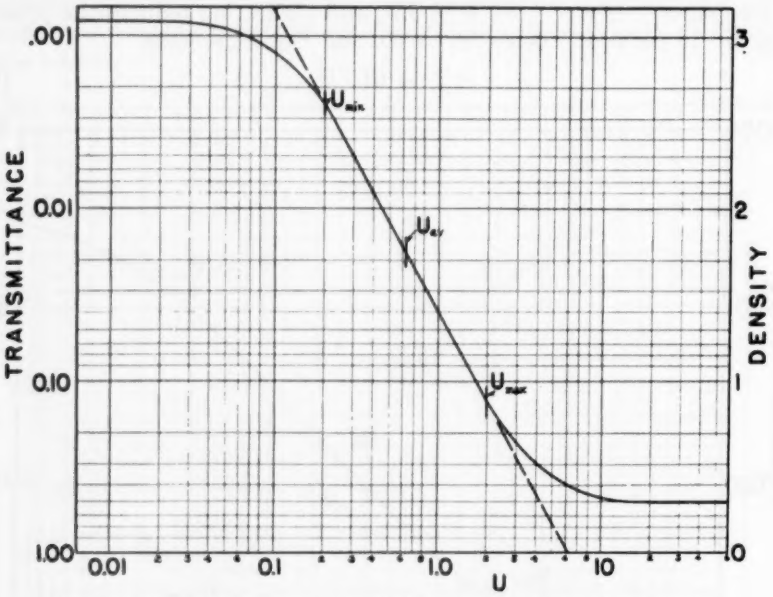

FIG. 5. Characteristic curve for a color transparency obtained by reversal.

For other films, characteristic curves can be obtained from the manufacturers (20) or by measurement. It is found that most negative films have characteristic curves whose shape is very much like that of Fig. 3 or 4. Figure 3 may be employed with other films as an approximation by merely shifting the U-scale in accordance with the film rating. A partial list is given in Table V.

Equation 9 gives a relation between the *camera exposure* t/f^2 and the ratio U/H. Consider a scene detail at point P (Fig. 1), the helios being H_P. This helios can be made to correspond to any value of U by varying t/f^2. The photographer should realize that the positioning of a given H on the characteristic curve is not done automatically: it is a conscious act requiring judgment. Once having fixed the relationship between any H and the corresponding U, however, we have fixed the camera exposure and the location on the characteristic curve of every value of scene helios.

TABLE V.—*Approximate Film Data.*

Film	U_{av} (lumen sec. m^{-2}) Daylight	Inc. lamps
Color (for daylight)		
Kodachrome, Professional Kodachrome, Ansco sheet film	0.63	
Ektachrome (roll and cut film)	0.85	
Ansco (toll and 35 mm)	0.50	
Kodacolor	0.25	
Color (for inc. lamps)		
Kodachrome A		0.42
Ektachrome B (professional sheet)		0.63
Ektacolor, Kodachrome B (professional sheet)		0.85
Kodacolor A		0.25
Black and White		
35-mm. films		
Super-XX, Ultra Speed Pan, Superior 3	0.04	0.05
Plus-X, Supreme, Superior 2, Panchromosa	0.08	0.10
Roll film		
Super-XX, Polaroid, Superpan Press	0.04	0.05
Verichrome, Plus-X, Supreme, Plenachrome, Superchrome, Panchromosa	0.08	0.10
Sheet film		
Tri-X Pan, Triple S Pan, Hi-Speed Pan	0.020	0.025
Super Panchro-Press	0.032	0.04
Ortho-X, Triple S Ortho, Ortho 7	0.032	0.05
Super-XX, Arrow Pan	0.04	0.05
Portrait Pan, Isopan, Superpan Portrait, XF Pan, Panchromosa	0.08	0.10
Panatomic X, FG Pan	0.10	0.21
Commercial Ortho	0.10	0.50
E. K. Commercial, Ansco Commercial, Defender Commercial	0.21	0.63

U_{max} is approximately 3.2 times the above values, U_{min} is approximately 0.32 of the above values.

For the negative films, $U_{max} \cong 10\ U_{av}$, $U_{min} \cong 0.1\ U_{av}$.

6. EXAMPLES

Table VI gives data on three widely different scenes which will be used as examples of exposure calculation. Data for other scenes are given elsewhere (13). Scene I was measured by Jones and Condit (8)

and the photograph is shown in the book of James and Higgins (21). The monument fills a large part of the picture, and is in direct sunlight with blue sky as background. Using the panchromatic film of Fig. 4 with a 4× green filter, we could vary the camera exposure by a factor of almost 10:1 and get excellent prints anywhere in this range. If $H = 5100$ blondel corresponds to $U_{min} = 0.01$, Eq. 9 gives for camera exposure

$$(t/f^2)_{min} = 6(0.01)4/5100 = 4.72 \times 10^{-5}$$

or

$$t = 0.0030 \text{ sec at } f = 8.$$

And if $H = 53{,}700$ blondel corresponds to $U_{max} = 1.0$,

$$(t/f^2)_{max} = 6(1.0)4/53700 = 4.46 \times 10^{-4}$$

or

$$t = 0.029 \text{ sec at } f = 8.$$

TABLE VI.—*Three Scenes.*

I. *Wright Memorial.* Clear day, front lighting, near view. Measurements by Jones and Condit (8).

Detail	H (blondel)	
Sky	16,100	
Stone, in sunlight	53,700	← Important range
Stone, in shadow	5,100	← 10.5:1
Grass	9,600	
Vines, av.	1,770	
Vines, min.	115	

II. *Moonlight on the Alps.* Clear sky, distant view (23).

Sky	0.015	
Snow, max.	0.20	← Important range
Snow, min.	0.02	← 10:1
Rocks in shadow	8×10^{-4}	
Lamp globe	20,000	
Lighted room	10	
Lamp-lighted snow	5	

III. *Library Reading Room.* Daylight fluorescent lamps behind ceiling of diffusing plastic, white walls, unstained-wood furniture.

Walls	1,530	
Ceiling	3,230	← Important range
Floor	1,070	← 3:1
Table tops	1,180	
Open book on table	1,530	
Shadows under tables	420	
Books in wall cases (min.)	45	

With Kodachrome, assuming the curve of Fig. 5, let us place the highest helios (53,700 blondel) at $U_{max} = 2.0$. Then

$$t/f^2 = 6(2.0)/53700 = 2.23 \times 10^{-4}$$

or

$$t = 0.007 \text{ sec at } f = 5.6.$$

Experience shows that this exposure would give an excellent Kodachrome picture of this subject. The exposure is also in agreement with exposure tables (22) which give 0.01 sec at $f = 5.6$.

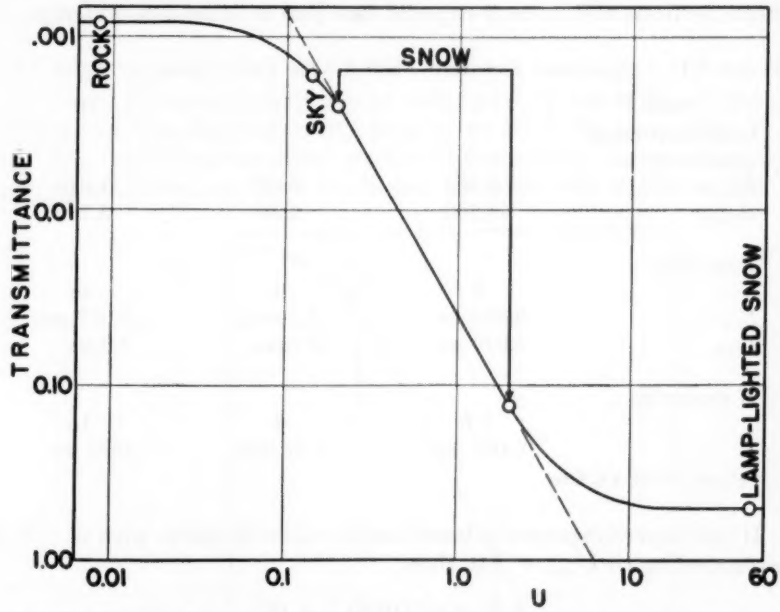

FIG. 6. Characteristic curve for a color transparency obtained by reversal. The film has been exposed 16 min. at $f = 4$ for Scene II of Table VI. The locations of the various parts of the scene are indicated by circles. Good gradation is obtained for the moon-lit snow, though other parts of the scene are necessarily over-exposed or under-exposed.

The second example is a distant snow scene taken by moonlight (23). The exposure should be based on helios measurements; but since none are available, a few calculations are necessary. It is well known (24) that the incident pharosage produced by a full moon is approximately 0.20 lumen m^{-2}. Since clean snow has a reflectance of almost 1.00, the maximum helios of the snow will be approximately 0.20 blondel. In the shadows, the snow will be illuminated only by the sky and will have about 1/10 of the maximum helios, just as it would in the daytime (Table II). The sky helios would also be about 0.02 blondel at ordinary

elevations; but since the picture is taken high in the mountains, the sky will appear a little darker, say 0.015 blondel. Rocks wiil have a reflectance of perhaps 0.04, so their helios (in shadow) will be 0.020 $\times$ 0.04 = 8 $\times$ 10^{-4} blondel. The artificially lighted details can be estimated from illuminating engineering practice (**24**).

The important part of the helios range is from 0.02 to 0.20 blondel, and a successful picture must exhibit good gradation over this range. The remainder of the details are unimportant. The rocks will be greatly under-exposed and will appear a uniform black, as indeed they would appear to an observer of the original scene. The artificially lighted portions will be over-exposed but that is of no consequence.

TABLE VII.—*Calculated Exposure Time for the Three Scenes of Table VI.*

Scene	I	II	III
Important range (blondels)			
H_{min}	5,100	0.02	1,070
H_{max}	53,700	0.20	3,230
*Pan. film**			
f	8	4	16
t_{min}	0.003 sec.	3.2 min.	0.057 sec.
t_{max}	0.029 sec.	32 min.	1.9 sec.
Kodachrome			
f	5.6	4	16
t	0.007 sec.	5–16 min.	0.95 sec.

* Figure 4 with 4$\times$ filter.

If the camera exposure is based on the moon-lit snow, with $H = 0.20$ corresponding to $U_{max} = 2.0$, then

$$t/f^2 = 6(2.0)/0.2 = 60$$

and

$$t = 960 \text{ sec (16 min) at } f = 4.$$

Location of details on the characteristic curve is shown in Fig. 6. Or if we attempt to bring the artificially lighted snow more nearly into range by moving $H = 0.20$ down to the middle of the straight part of the characteristic ($U_{av} = 0.63$),

$$t/f^2 = 6(0.63)0.2 = 19$$

or

$$t = 5.0 \text{ min at } f = 4.$$

Windisch (**23**) obtained a very satisfactory picture with an exposure time of 7 min. at the above aperture.

The third example is of a library reading room, artificially lighted by a modern luminous ceiling (25). The values of helios may be obtained by measurement, or they may be calculated by the methods used in designing lighting systems. The important range in this example is from 1070 to 3230 blondels, the latter being the helios of the ceiling, which is the brightest thing in the room. If we place $H = 3230$ at $U_{max} = 2.0$, we obtain

$$t/f^2 = 6(2.0)/3230 = 3.71 \times 10^{-3}$$

or $t = 0.95$ sec at $f = 16$ for Kodachrome. Other exposure times for the three examples are listed in Table VII.

7. A SLIDE RULE

Camera exposure can always be calculated by use of Eq. 9, though one may have a slight difficulty keeping in mind the two values H_{max} and H_{min} and their locations on the characteristic curve. To allow easy visualization, we have developed the slide rule shown in Fig. 7.

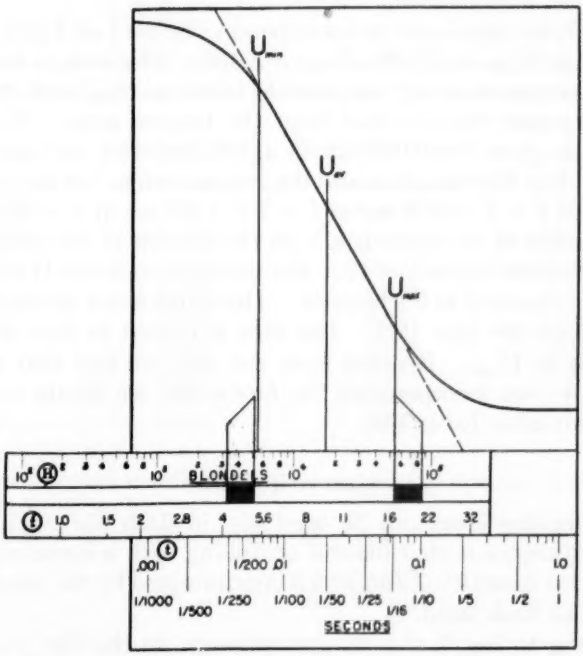

FIG. 7. Slide rule for determining exposure by the two-helios method. The two indices are first set at the measured values of H_{max} and H_{min}. The slide (which carries the indices) is then moved to right or left until the indices fall within a desirable part of the characteristic. The exposure time t for any f-number is then read off the bottom scale.

The characteristic curve of the film is shown at the top of the device. If several films are to be used, their curves can be included; or detachable graphs may be used. For a given t and f, U is directly proportional to H,

$$U = (t/6f^2)H,$$

so there is no need to include a scale of U.

The helios scale is a logarithmic scale exactly like the U-scale of Fig. 5, but it is mounted on a slide so that it can be moved with respect to the graph. The fixed t-scale is also exactly like the U-scale. The f-scale is logarithmic but is spread over twice the distance because f occurs in Eq. 9 as the square. The scales can be obtained very easily by cutting strips from a piece of logarithmic cross-section paper. Two indices of red plastic are movable along a slot that is machined in the slide. These indices are set at H_{max} and H_{min} for the scene. The slide is then moved until the indices fall on a part of the characteristic that seems best to the photographer. The exposure time for any f-number is read off the bottom scale.

In Fig. 7, the rule is set to correspond to Scene I of Table VI. The indices are at $H_{max} = 53{,}700$, $H_{min} = 5{,}100$. The slide is moved until this range comes as nearly as possible between U_{max} and U_{min} of the graph. Exposure time is read from the bottom scale. For $f = 5.6$, the slide rule gives $t = 0.007$ sec, in agreement with the calculation of Section 6. But the rule gives also the exposure time, for any other stop: 0.0035 sec at $f = 4$, 1/500 sec at $f = 2.8$, 1/10 sec at $f = 22$, etc.

If the helios of the scene is not on the H-scale of the slide rule, one merely introduces a power of 10. For instance, in Scene II of Table VI, the helios of the snow in 0.2 blondel. This value is not on the rule, so we multiply it by 10^n (say 10^5). The slide is moved so that $H = 20{,}000$ corresponds to U_{max}. Reading from the rule, we find that $t = 0.0096$ sec at $f = 4$; and incorporating the factor 10^5, we obtain $t = 960$ sec, in agreement with Table VII.

8. FLASH PHOTOGRAPHY

The foregoing ideas can be used also in flash photography. The principal difference is that instead of dealing with a constant helios H, we deal with a quantity $\int H dt$ which is determined by the total luminous output of the flash lamp.

According to Eq. 9, the incident phosage on the film, caused by a single flash, is

$$U = \frac{1}{6f^2}\int H_P dt = \frac{\rho_P}{6f^2}\int D_P dt, \qquad (10)$$

where

D_P = incident pharosage (lumen m^{-2}) on the scene at P,

$\int D_P dt$ = incident phosage (lumen sec m^{-2}) on scene at P,

ρ_P = reflectance of scene detail at P.

Let Q = total phos (lumen sec), output of the flash lamp. For a source that radiates uniformly in all directions, with no reflector and no interflections within the room,

$$\int D_P dt = Q \cos\theta / 4\pi l^2,$$

where l = distance (m) from flash lamp to subject, and θ = angle of incidence. Ordinarily a reflector is used, resulting in an increased pharosage on the subject. Then

$$\int D_P dt = MQ \cos\theta / 4\pi l^2, \tag{11}$$

where M is a parameter that depends on the size and shape of the reflector, on its reflectance and specularity, and on interflections in the

TABLE VIII.—*Photoflash Lamps.*

Designation	Color temp., °K.	Approx. time to reach peak, sec.	Q, lumen sec.
SM	3,300	0.006	4,700
5	3,800	0.021	16,000
5B	6,000	0.021	7,000
11	3,800	0.021	29,000
22	3,800	0.021	63,000
22B	6,000	0.021	27,000
50	3,800	0.030	93,000
50B	6,000	0.030	40,000

room. For average conditions with a small (12 cm. dia.), matt aluminum reflector, $M \cong 6$.

Substitution of Eq. 11 into Eq. 10 gives

$$l^2 f^2 = (MQ/24\pi) \frac{\rho_P \cos\theta}{U}. \tag{12}$$

This relation plays a role in flash photography similar to that played by Eq. 9 in regular photography. Equation 12 relates the phosage U incident on the film to the reflectance ρ_p of the scene detail.

To obtain a simple formula for practical use, replace quantities at a

point by *average* quantities and write

$$f = \frac{1}{l}\left[(MQ/24\pi)\rho_{av}\cos\theta/U_{av}\right]^{\frac{1}{2}}. \qquad (12a)$$

The quantity $\left[(MQ/24\pi)\rho_{av}\cos\theta/U_{av}\right]^{\frac{1}{2}}$ is called the *guide number.* It is tabulated (22) as a function of lamp and film; so all the amateur need do in calculating the aperture is to divide guide number by distance. Equation 12*a* is better than the ordinary tables, for it allows the use of the correct values of ρ_{av}, M, and θ.

For example, a photograph is to be taken on Kodachrome with a No. 5B lamp (Table VIII). Assuming an average reflectance of 20 per cent for the subject, we have $Q = 7000$ lumen sec, $\rho = 0.20$, $U_{av} = 0.63$ (Fig. 5), $M = 6$. Thus

$$\left[(MQ/24\pi)\rho_{av}/U_{av}\right]^{\frac{1}{2}} = \left[\frac{6(7000)}{24\pi}\frac{0.20}{0.63}\right]^{\frac{1}{2}} = 13.3,$$

which is the guide number in mks. units. If l is expressed in feet instead of meters, the guide number is $13.3 \times 3.28 = 44$, which agrees with the tabulated value (22) of 45. The synchronized shutter must be set at 1/25 sec or more, otherwise the total phos of the lamp will not be utilized and a correction will be necessary.

TABLE IX.—*Electronic Flashtubes.*

No.	C, μf	V, volt	$\frac{1}{2}CV^2$, watt sec	Q, lumen sec.
FT–210 FT–214	100	2,000	200	7,000
FT–403	240	2,000	480	18,000
FT–503	250	4,000	2,000	100,000

Flashlight with electronic tubes is calculated exactly like that for photoflash lamps. The basic difference in the luminous output of the two is that the flashtube gives a much shorter flash, lasting approximately 0.3 millisecond instead of 15 milliseconds. This means that the shutter setting has no effect on the film phosage produced by the flash. Some xenon flashtubes are listed in Table IX. Note that the Q of the tubes is comparable with that of photoflash lamps, though the peak pharos (lumens) is much greater because of the shorter flash. The color of the light is similar to daylight, so daylight color film is used and daylight film indices are employed (Table V).

It might seem that Eq. 12*a* would constitute a very inaccurate way of determining exposure, in as much as no attempt is made to include the helios ratio of the subject. But the important point is that the lamp is mounted on or near the camera so that the lighting is very flat. Shadows are almost eliminated and *the helios range is determined prin-*

cipally by the range of reflectances found in the subject. The highest reflectance ordinarily encountered is 0.80 and the lowest is usually not below 0.04, so the helios range is much less than in some naturally lighted outdoor scenes. By setting $\rho_{av} = 0.20$ for the geometric mean, we obtain a reflectance range from 0.063 to 0.63 for the 10:1 range of Kodachrome.

TABLE X.—*Representative Reflectances.*

(For 7000° Planckian radiation)

Material	ρ
Paints, new	
White	0.90
Ivory	0.75
Cream	0.67
Yellow	0.70
Pink	0.55
Green	0.37
Blue	0.11
Brown	0.11
Dark red	0.07
Wood, unstained	
Prima vera	0.42
White oak	0.26
Zebrawood	0.20
Mahogany	0.13
Walnut	0.13
Oak, stained	0.12
Masonite	0.09
Carpets	
Tan	0.11
Red	0.06
Dark brown	0.03
Dark green	0.019
Dark blue	0.015
White paper	0.80
Human skin (untanned Caucasian)	0.40
Hair, brunette	0.08
blonde	0.15

For the average flashlight picture, therefore, ρ_{av} may be set at 0.20 in Eq. 12*a*, and $U_{av} = 0.63$ for Kodachrome. For very light or very dark subjects, however, the proper value of ρ_{av} should be used. Table X may be found helpful. For instance, in taking a Kodachrome picture of a Scottish terrier we might use $\rho_{av} = 0.02$ to get good gradation in the blacks. Blind adherence to $\rho_{av} = 0.20$ would certainly produce gross

under-exposure in this case. For a portrait of a white cat, on the other hand, $\rho_{av} = 0.70$ would be better than $\rho_{av} = 0.20$.

Now consider the use of *two lamps* arranged to flash simultaneously (**26**). A single lamp at the camera tends to produce rather uninteresting portraits since there are no shadows. But if the lamp is moved to give side lighting, the shadows are too dense, the helios ratios being perhaps 20:1. A pleasing gradation is obtained with a 3:1 helios ratio (**27**), helios of the highlights on the face being 3 times the helios of the face shadows. The desired ratio is obtained by using a second lamp to fill in the shadows. The principal lamp may be located about 45° to the side of the lens axis and perhaps 45° above it, so that it simulates sunlight. The second lamp is placed near the lens axis.

Let $\mu = U_{1D}/U_{2D} = H_{1D}/H_{2D}$, $\nu = U_1/U_2 = H_1/H_2$, where subscript 1 refers to the highlight, subscript 2 to the shadow, D indicates the principal lamp, F the "fill-in" lamp. The two lamps and reflectors are assumed to be identical. Thus μ is the helios ratio with one lamp, while ν is the improved ratio obtained by using two lamps. Then

$$\left.\begin{aligned} (l_F/l_D)^2 &= \frac{\mu(\nu \cos\theta_{2F} - \cos\theta_{1F})}{(\mu - \nu)\cos\theta_{1D}}, \\ f &= \frac{1}{l_D}\left[\frac{\rho\nu\cos\theta_{1D}}{\mu U_1}\left(\frac{MQ}{24\pi}\right)\frac{\mu\cos\theta_{2F} - \cos\theta_{1F}}{\nu\cos\theta_{2F} - \cos\theta_{1F}}\right]^{\frac{1}{2}}. \end{aligned}\right\} \tag{13}$$

The first equation gives the ratio of the lamp distances required for the desired modeling; the second equation is a modified guide-number equation giving the stop that must be employed. For portraiture, we may take $\cos\theta_{1F} = \cos\theta_{2F} = \cos\theta_{1D} \cong 0.8$, so Eq. 13 becomes

$$\left.\begin{aligned} (l_F/l_D)^2 &= \frac{\mu(\nu - 1)}{\mu - \nu}, \\ f &= \frac{1}{l_D}\left[\frac{0.8\rho\nu}{\mu U_1}\left(\frac{MQ}{24\pi}\right)\frac{\mu - 1}{\nu - 1}\right]^{\frac{1}{2}}. \end{aligned}\right\} \tag{13a}$$

For instance, if $\mu = 20$, $\nu = 3$, $\rho = 0.4$, $(MQ/24\pi) = 557$, $U = 0.63$, Eq. 13*a* gives

$$l_F/l_D = 1.5, \quad f = 20/l_D.$$

Thus a pleasing gradation will be obtained if the fill-in lamp is 50 per cent further from the subject than the main lamp. The guide number is 20 m.

As a final example of flash photography, consider the combination of *daylight and electronic flash*. With daylight alone, the helios ratio is found to be $\mu = H_{1D}/H_{2D}$. This ratio may be changed to the desired ratio ν by adding a single flashtube near the lens axis.

It can be shown that

$$\left.\begin{aligned} l_{02}^2 &= \frac{(MQ/24\pi)}{f^2 U_2(\mu - \nu)} \left[\mu\rho_2 \cos\theta_2 - k\rho_1 \cos\theta_1\right], \\ t &= \frac{6f^2 U^2 \mu \left[\nu\rho_2 \cos\theta_2 - k\rho_1 \cos\theta_1\right]}{H_{1D} \left[\mu\rho_2 \cos\theta_2 - k\rho_1 \cos\theta_1\right]} \end{aligned}\right\} \tag{14}$$

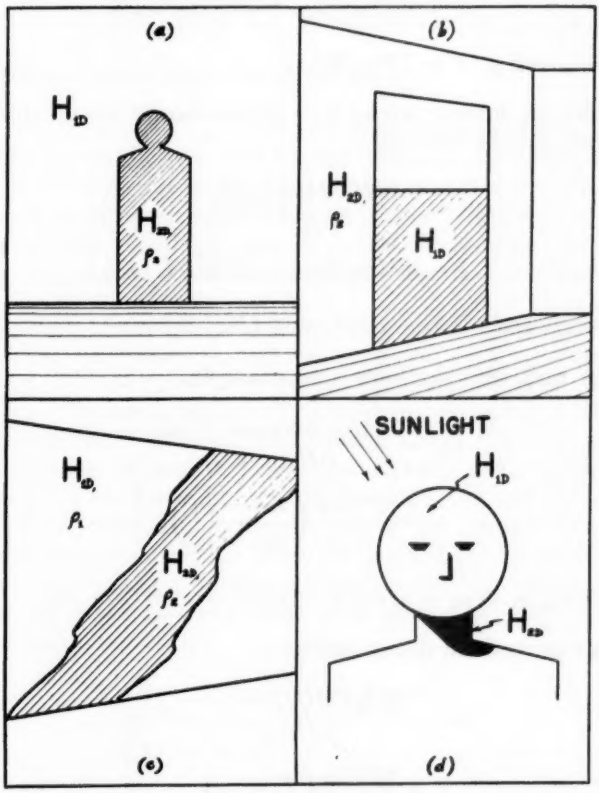

FIG. 8. Schematic diagrams for four combinations of daylight and flashlight.
(*a*) Near-by object with distant background.
(*b*) Interior view which includes outdoor scene.
(*c*) Shadow on a wall.
(*d*) Portrait in direct sunlight.

The first of these equations determines the distance l_{02} between flashtube and principal subject; the second equation fixes the shutter setting. Here $k = (l_{02}/l_{01})^2$ and l_{01} = distance between flashtube and object No. 1.

Four special cases are indicated in Fig. 8.

(I) $l_{01} \gg l_{02}$, $\cos\theta_2 = 1$, $k \to 0$ *(Fig. 8a)*

Here the background is so far away that its helios is unaffected by the flashtube. The general equation then reduces to

$$\left.\begin{aligned} l_{02}^2 &= \frac{\mu\rho_2(MQ/24\pi)}{f^2U_2(\mu-\nu)}, \\ t &= 6f^2U_2\nu/H_{1D} = 6f^2U_1/H_{1D}. \end{aligned}\right\} \tag{14a}$$

(II) $l_{01} \gg l_{02}$ *and* $\mu \to \infty$ *(Fig. 8b)*

Examples are interior views that include sunlit scenes through windows. Then Eq. 14 becomes

$$\left.\begin{aligned} l_{02}^2 &= \frac{\rho_2(MQ/24\pi)\cos\theta_2}{f^2U_2}, \\ t &= 6f^2U_2\nu/H_{1D} = 6f^2U_1/U_{1D}. \end{aligned}\right\} \tag{14b}$$

(III) $l_{01} \cong l_{02}$ *and* $\cos\theta_1 = \cos\theta_2 = 1$ *(Fig. 8c)*

A sunlit wall with shadows is an example.

$$\left.\begin{aligned} l_{02}^2 &= \frac{(MQ/24\pi)(\mu\rho_2-\rho_1)}{f^2U_2(\mu-\nu)}, \\ t &= \frac{6f^2U_2\mu}{H_{1D}}\left(\frac{\nu\rho_2-\rho_1}{\mu\rho_2-\rho_1}\right). \end{aligned}\right\} \tag{14c}$$

(IV) $l_{01} \cong l_{02}$, $\rho_1 = \rho_2 = \rho$, $\cos\theta_1 = \cos\theta_2 = 0.8$ *(Fig. 8d)*

Outdoor portrait in direct sunlight.

$$\left.\begin{aligned} l_{02}^2 &= \frac{0.8\rho(MQ/24\pi)(\mu-1)}{f^2U_2(\mu-\nu)}, \\ t &= \frac{6f^2U_2\mu}{H_{1D}}\left(\frac{\nu-1}{\mu-1}\right). \end{aligned}\right\} \tag{14d}$$

As an example, take a picture in a room with a door opening on a sunlit garden. Measurement shows that the average helios of the garden is $H_{1D} = 6000$ blondel, while the average helios of the room walls is $H_{2D} = 20$ blondel. The ratio μ is so large that we may use Eq. 14*b*. Let $\nu = 3$ to make the interior look reasonably dark. Then for daylight Kodachrome and an *FT*-214 tube operated with 100 μf and 2000 volts, $M = 6$, $Q = 7000$ lumen sec, $U_2 = 0.4$, $\rho_2 = 0.30$, $\cos\theta_2 = 1.0$, $\nu = 3$, $f = 5.6$, and

$$\left.\begin{aligned} l_{02} &= \left[\frac{0.30(557)}{0.4(5.6)^2}\right]^{\frac{1}{2}} = 3.6 \text{ m}, \\ t &= 6(0.4)3(5.6)^2/6000 = 0.04 \text{ sec}. \end{aligned}\right\}$$

9. SUMMARY

The practical results on photographic exposure may be summarized in the simple equation,

$$t/f^2 = 6U_Q/H_P. \tag{9}$$

This equation relates the helios H_P of a scene detail (Fig. 1) to the incident phosage U_Q on the film. Knowing the characteristic curve for the film, the photographer decides what value of U_Q is to correspond to a given value of H_P. Then Eq. 9 determines the camera exposure (t/f^2).

The photographer makes two decisions:

(*a*) What are H_{max} and H_{min} *for the range in which gradation is desired?* These limits rarely coincide with the true maximum and minimum helios of the scene: they depend on what the photographer wishes to emphasize and what he wishes to ignore.

(*b*) Where should the U_{max} and U_{min} (corresponding to H_{max} and H_{min}) be placed on the characteristic curve of the film?

We advocate the *two-helios method* of calculating camera exposure. H_{max} and H_{min} are measured, and these two values are related to U_{max} and U_{min} on the characteristic curve. Equation 9 then determines exposure time t for any aperture f. The slide rule of Fig. 7 allows easy visualization of the relations. The constant 6 in Eq. 9 is an average value based on representative lens transmittance and a point 15° from the lens axis. This constant may be re-evaluated for the particular lens being used, though such refinement is not ordinarily necessary. One should also remember that when photographing at short distances, the f of Eq. 9 is not the number engraved on the lens barrel.

Various *one-helios methods* have been advocated (**28**) to simplify the calculation of exposure:

(1) Calculation based on H_{av},
(2) On H_{min},
(3) On H_{max}.

These schemes are inferior to the two-helios method because they do not allow the photographer to visualize how his highlights and shadows will be reproduced.

In (1), some sort of average helios is measured for the scene, and the camera exposure is

$$t/f^2 = 6U_{av}/H_{av}. \tag{9a}$$

If H_{av} is the geometric mean of H_{max} and H_{min}:

$$H_{av} = \sqrt{H_{max}\,H_{min}},$$

and if both H_{max} and H_{min} are known and their ratio H_{max}/H_{min} has a value that can be reproduced, then Eq. 9*a* offers a good method of calculating exposure. But if H_{av} is taken as the reading of an ordinary photoelectric exposure meter, the results are practically worthless, as has been shown by Jones and Condit (**8**) and by Berg (**14**).

For (2), the film is exposed for the shadows:

$$t/f^2 = 6U_{min}/H_{min}. \tag{9b}$$

Used without additional information, this method tells nothing about what will happen to the highlights. For (**3**),

$$t/f^2 = 6U_{max}/H_{max}, \tag{9c}$$

which tells nothing about the shadows. These schemes are usable, but only if the ratio H_{max}/H_{min} is known to be within allowable limits. The paper concludes with a consideration of flash exposure based on the same methods.

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Soap Bubbles in Metallurgical Research.—Study of soap bubbles by scientists in the General Electric Research Laboratory may aid in the development of metals that are stronger than those now in use and that have other improved properties. Such bubbles are helping to explain fundamental data on the behavior of metals.

According to Dr. Robert L. Fullman, research associate in the Laboratory's Metallurgy Research Department, the soap bubbles resemble in many respects the crystals or grains of which all metals are made. In particular, he has found, the way that little bubbles grow into big ones is closely analogous to the growth of metallic grains.

Neither the bubbles nor the grains ever grow by the coalescence of two smaller units into a larger one, he said. Instead, when a bubble or grain gets bigger, its boundaries expand at the expense of adjacent ones which contract and finally disappear.

Dr. Fullman's bubbles are not blown in the open air, but in special glass cells about five inches in diameter and half an inch thick. Each cell is half-filled with a special soap solution. It consists of a liquid sold in toy stores for making bubbles, to which other chemicals have been added to improve the performance. Then the air is pumped out of the cell and the space above the liquid becomes filled mainly with water vapor. The tube to the vacuum pump is sealed off, so the exhausted cell may be handled.

When the cell is shaken vigorously and then laid on a flat surface, thousands of tiny bubbles appear above the liquid. After it has been allowed to stand a little while (ten or fifteen minutes) the bubbles are larger and fewer. At this stage their continued growth may easily be observed. Dr. Fullman has also made lapse-time motion pictures which show the movements speeded up considerably.

The bubbles have varying numbers of sides, but when there are only three sides on a bubble, it starts to disappear. The three sides shrink, while the vapor inside migrates through the walls into adjacent bubbles, which are enlarged accordingly.

Dr. Fulman says that in a single metal grain the atoms are lined up like bricks in a wall. So are the atoms in a grain next-door, but the rows in one grain do not line up with those in the other. The line of discontinuity is the boundary between the grains.

When a metal is heated, some of the grains enlarge, while others shrink and disappear, just as in the soap bubbles, he said. As the boundary of a grain passes an atom, the atom shifts its position a little to get into line with the rows in the expanding crystal.

Many metallurgical applications, such as the steel used in electrical transformers, depend on accurate knowledge and control of metal grains, Dr. Fullman pointed out. Hence, studies of their behavior, by the bubble technique and other means, are expected to lead to new knowledge which may greatly improve the performance of metallic structures, he said.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS *

NBS PHYSIOLOGICAL MONITOR

Working under the sponsorship of the Veterans' Administration, the National Bureau of Standards has developed an electronic instrument which automatically detects changes in the physiological condition of a patient under anesthesia throughout the course of an operation. Known as the NBS Physiological Monitor, this instrument measures changes in the patient's blood pressure, heart beat, and respiration as they occur and presents the data on a panel for interpretation by the surgeon or anesthesiologist. A permanent record of the patient's condition during the operation is also provided by a recording device incorporated in the assembly.

Because the Physiological Monitor provides a continuous indication of the patient's physiological condition during surgery, it should prove of great value in the prevention and control of emergencies which may unexpectedly confront the surgeon at the operating table or during certain critical postoperative periods. The instrument is also expected to be a distinct aid in those phases of medical research—such as studies of the effect of drugs on blood pressure—that require a knowledge of the behavior of physiological variables over long periods.

Development of the Physiological Monitor was carried out by S. R. Gilford and H. P. Broida of the NBS staff. Financial support was provided by the Veterans' Administration and by a program of basic instrumentation sponsored at NBS by the Office of Naval Research, the Office of Air Research, and the Atomic Energy Commission. Though the present equipment was designed and constructed primarily to evaluate the feasibility of electronic instrumentation as an aid to operating room procedure, preliminary trials in medical wards at Mount Alto Veterans' Administration Hospital (Washington, D. C.) and in the operating room at George Washington University Hospital have indicated great potential usefulness to the surgical team.

During surgery it is vital that the condition of the patient be known at all times. Ordinarily the anesthesiologist measures blood pressure and pulse rate at sufficiently close intervals to maintain a satisfactory evaluation of the patient's condition. If necessary, he administers drugs, additional anesthetics, or infusions of blood or plasma. However, when the anesthesiologist is occupied with his other duties, longer periods may elapse between measurements. Although in most cases this delay is not serious, at other times it can result in unfortunate

* Communicated by the Director.

consequences. For example, when complete circularory collapse occurs —as in severe blood loss or in heart failure—there may be some lapse of time before the condition becomes outwardly apparent. Since time is important in applying remedial measures, the earliest possible detection of such a crisis offers the best hope in saving the life of the patient.

The NBS Physiological Monitor makes prompt emergency action possible by continuously displaying the information the anesthesiologist needs to know in simple, numerical form. Data on systolic and diastolic blood pressure,[1] pulse rate and pulse irregularity, breathing rate, and amount of air breathed out per minute are available at a glance without intermediate calculation or manipulation. Preliminary adjustments are easily made, requiring no special technical skill or training on the part of the operator. Safeguards are provided to permit the instrument's use in the high combustible mixtures of anesthetic gases that are usually present in the operating room. Size and weight of the equipment have been minimized for easy moving, and the amount of floor space required has been kept as small as possible.

While individual systems were already in existence for performing some of the functions of the Physiological Monitor, for other functions it was necessary to develop entirely new methods of approach. In general, the problem was to integrate the various component measuring and indicating systems into a complete instrument which would function smoothly and safely in the operating room which would present results in a clear, easily interpreted manner.

All of the electronic assemblies used in the Physiological Monitor were made so as to be easily replaceable, and the "plug-in" type of structure was chosen to facilitate servicing. Sufficient flexibility was built into the present experimental model to permit the inclusion of instrumentation for measuring additional physiological quantities as further developmental work is done on the instrument.

General Description

The Physiological Monitor is housed in two metal cabinets—the *operating room console* and the *recorder console*—interconnected by a combination electrical cable and air line permitting a maximum separation of about 20 feet.

The operating room console, which has been rendered safe for use in the presence of explosive vapors, is three feet high and occupies 22 by 26 inches of floor space. It provides a compact, easily read display of the results of all measurements as they are made. This unit also contains the controls used to place the instrument in operation. All attachments to the patient are made from this unit, which houses most of the measuring equipment.

[1] Systolic pressure is associated with the heart's contraction, diastolic pressure with heart expansion.

The recorder console, 5 feet high and occupying 22 by 24 inches of floor space, is designed to be placed outside of the operating room, where explosive vapors are not present. This unit contains a recorder which provides a permanent record of the measurements indicated on the operating room console, additional circuit components, and an air supply for use in blood pressure measurements and for pressurizing the operating room console to keep it free of explosive vapors.

Blood Pressure Measurement

The automatic system used in the Physiological Monitor for measuring blood pressure is based upon the technique commonly used by an examining physician. The physician forces air into a hollow band, or cuff, surrounding the arm until the systolic pressure is exceeded; he then gradually reduces the pressure and determines systolic and diastolic pressures by noting the pressure reading on a manometer when certain characteristic pulse sounds in the artery are detected with a stethoscope.

In the automatic system, a microphone is located at the point of observation over the brachial artery. Every three minutes the valve of an air supply automatically opens, allowing the pressure within the arm band to increase. As soon as the pressure in the band exceeds the diastolic value, the microphone begins to pick up sound within the artery. This sound reaches a maximum and decreases, disappearing after the pressure in the band exceeds the systolic point. By means of a system of amplifiers and relays, the pulses picked up in the microphone actuate two solenoid valves in the air system which open to connect the system to the proper pressure indicating gages at the diastolic and systolic points. The valves close almost immediately after opening so that the diastolic and systolic pressures—now converted by a transducer to electrical signals—remain registered on the indicating meters until the next measurement cycle begins.

The amplifier and relay circuit are arranged in such a way that when the microphone receives the initial sound from the artery, the first valve opens, permitting the adjacent gage to register the diastolic pressure. Since it is not possible to determine the point at which sound disappears until that point has been passed, the pressure is now carried beyond the systolic point and then allowed to decrease slowly until the systolic pressure is again reached. As the first sounds corresponding to the systolic pressure are received, the second valve opens, allowing its adjacent gage to "capture" and retain the instantaneous pressure of the system. The band is then rapidly deflated to minimize the period during which pressure is applied to the patient's arm.

The entire process proceeds under the control of a program switch which initiates the measurement at three-minute intervals. The actual determination takes about one minute or less for the normal range of

pressure but is dependent upon the maximum pressure the system must reach. The relay system selects the proper pressure registration valve corresponding to the systolic or diastolic measurement and permits only the first sound pulse to operate the registration valve. The relay system also starts the deflation of the arm band after the systolic pressure has been exceeded by a safe margin (as indicated by the cessation of sound) and finally opens a second valve which rapidly deflates the band after the systolic pressure has been recorded.

The microphone-amplifier combination selects sound pulses of the required amplitude and frequency content, and converts them into electrical pulses of uniform amplitude and duration which actuate the relay and control circuits. Because the sound originating in the artery is considerably attenuated by the time it emerges from the tissues, the microphone-amplifier circuit must be quite sensitive to low-amplitude pulses. At the same time it must minimize unwanted signals originating from microphone frictional noises and random sounds in the immediate vicinity. By improvement of the microphone, by shaping of the amplifier pass band characteristics to the restricted band of signals, and by development of special coincidence and verification circuits, it was possible to construct a sensing device which sharply outlines the end points and is virtually undisturbed by spurious signals.

A crystal microphone with good response at the lower audio frequencies was incorporated into a standard stethoscope chest piece. The microphone is thoroughly shielded to keep electrical noise out of the amplifier input. The output of the microphone is applied to a three-stage conventional RC-coupled amplifier having a gain of approximately 80 decibels. This is followed by a low-pass filter which starts to cut off above 100 cps.

The output of the filter is introduced into a pulse stretching and equalizing circuit in the form of a monostable multivibrator. A neon tube connected to the plate of the normally nonconducting triode of the multivibrator flashes each time a pulse triggers the multivibrator. This neon tube, located on the instrument panel, serves as an indicator which permits the operator to adjust the gain in the system to the characteristic sound of the subject.

The system for discrimination between the sound produced within the artery and other noise signals is based upon the fact that the sounds within the artery are accompanied by corresponding slight increases in pressure within the arm band. The coincidence circuits reject all sound pulses that do not coincide with the necessary pressure pulses. As a further protection, these circuits prevent actuation of the relay system by any sound pulse that is not followed by a similar pulse within $\frac{1}{2}$ second.

The blood pressure measurement system has been used on many different human subjects for periods ranging from an hour to as long as

21 hours under hospital conditions. In general, it appears to cause the patients no discomfort and permits them to eat, sleep, and carry on other activities of a bed patient. Good agreement has been found between instrument values of blood pressure and those obtained in the usual way. Changes in pressure following the administration of drugs affecting blood pressure are easily followed.

Measurement of Pulse Rate

It is important to monitor the pulse rate during anesthesia, since the action of some of the anesthetic drugs and the effect of surgery tend to increase cardiac irritability. The system used in the Physiological Monitor to measure pulse rate is a modification of a method described by Sturm and Wood.[2] While the physician ordinarily checks heart beat, or pulse rate, by counting the pulsations of an external artery, this method obtains the information by measuring the periodic changes in the very small voltage produced by the heart. Electrical connections are made to the patient's arms and legs as in the use of the electrocardiograph for study of heart conditions.

By means of suitable circuitry, the time between adjacent pulses in the cardiac potential is continuously measured and converted to rate on a specially calibrated meter. Thus the instantaneous heart rate is measured, rather than the average rate over a given period. This makes it possible to follow irregularities and abrupt changes in heart beat.

The time intervals between heart beats are made proportional to the voltage across a linearly charged capacitor. Each cardiac pulse discharges the condenser to zero, after which the voltage begins its linear increase. The peak voltage reached at the time of each pulse is stored in a capacitor memory to which the first capacitor transfers its charge.

The input from the patient electrodes is applied to a differential amplifier input stage, which is adjusted to give good discrimination against electrostatically induced interference from the power line. The resulting output is amplified by several additional stages and is applied finally to a pulse shaper in the form of a monostable multivibrator. The signal level is adjusted to trigger the multivibrator once each cardiac cycle. This causes the momentary actuation of a relay which transfers the voltage across the timing capacitor to the storage capacitor and then resets the timing capacitor to zero voltage. An indicator lamp flashes in synchronism with the pulse, providing a convenient means for obtaining the patient's pulse rate by the conventional method if desired.

Once the electrodes are adjusted, the operator has only to read the

[2] "An Instantaneous Recording Cardiotachometer," by R. E. Sturm and E. H. Wood, *Rev. Sci. Inst.*, Vol. 18, No. 10 (October, 1947).

values of pulse rate from the dial. Continuous measurements have been made on medical ward patients for as long as 21 hours without difficulty.

Measurement of Cardiac Irregularity

Arrhythmia, or cardiac irregularity, may be the forerunner of serious difficulties for the patient in the operating room. However, objective measurement of arrhythmia is complicated by the fact that both changes in rate and occasional irregular beats occur under normal as well as abnormal conditions. The system developed by the Bureau was therefore designed to weight the rate of occurrence of such "arrhythmic incidents" rather than to indicate these events individually.

The system operates by comparing the time intervals between adjacent pairs of beats on a continuous basis. Time differences of 25 per cent are arbitrarily defined as arrhythmic incidents. The circuit gives the average time rate of these occurrences and presents it on an arbitrary scale.

This measurement is accomplished rather simply through use of the circuit developed for measuring pulse rate. If the heart beats are uniformly spaced, then only enough charge will be transferred to the memory capacitor to replace its losses, which are relatively small. However, should there be a considerable discrepancy between beats, then the memory capacitor has to receive or give up enough charge to bring it to the required level. In this process current flows, and if the magnitude of this current exceeds a certain value, it is used to indicate an arrhythmic incident.

Each time a pulse of current flows to the storage capacitor it passes through the primary winding of a transformer. The voltage pulses from the secondary are introduced into a pulse height discriminator. Whenever a pulse is of sufficiently high amplitude, the discriminator produces an output pulse that is fed into an integrator circuit. The time average of these pulses appears on the output indicator. For purposes of studying the system's response to arrhythmia, a circuit is also included which automatically turns on an electrocardiograph whenever unusual irregularity of heart beat is indicated.

Measurement of Respiratory Rate and Volume per Minute

While measurements of respiratory function are not ordinarily available during an operation, such information should be of considerable value. Under general anesthesia those parts of the brain which normally control respiration become depressed to an extent depending on the depth of anesthesia. Also, some surgical procedures involving the thoracic cavity are accompanied by mechanical impediments to breathing. Development of a system for obtaining information on respiration rate and volume should permit the routine application of

these measurements during surgery and should also provide a means for determining their value to the anesthesiologist.

The basis of the respiration measuring equipment in the Physiological Monitor is a positive displacement flowmeter in which the rotation of the output shaft is proportional to the volume of gas that has passed through the meter. For measurement of volume of flow per minute, the shaft of the flowmeter is periodically coupled to an indicating system for one minute. The accumulated volume indicated for this time is preserved until the next determination by a memory system consisting of a synchronous motor-generator pair.

During each one-minute registration period, the shaft of the flowmeter is coupled through a clutch to the synchro-generator, and the synchro-generator is electrically disconnected from the synchro-motor. At the end of the one-minute measuring interval, the generator is declutched from the flowmeter shaft, its shaft clamped, and electrical connections completed to the synchro-motor. The motor shaft immediately assumes an angular position corresponding to that of the generator shaft. The electrical connection between generator and motor is then broken, so that the motor shaft retains the angular information transferred from the synchro-generator. The motor shaft carries with it the movable arm of a potentiometer which translates angular position into a voltage suitable for meter indication and recording. At the end of the cycle, the generator shaft is brought back to its original starting point and again coupled to the flowmeter.

Respiration rate is determined as the number of times the shaft of the flowmeter starts and stops with a given time as the patient breathes in and out. The motion of the shaft is followed by means of an attached multitoothed wheel. As the shaft rotates, this wheel alternately opens and closes a pair of electrical contacts. Each actuation of the contacts discharges a capacitor which controls the potential of a trigger circuit. As long as the contacts continue to open and close, the control potential is held to a value below the triggering threshold. But as soon as contact actuation is interrupted for a time indicating momentary cessation of motion, the trigger operates and sends a pulse into a register. Each pulse thus indicates one respiration.

The pulse count is accumulated on a 50-point telephone-type stepping switch, where it is converted into the angular displacement of the switch shaft. This shaft is coupled to a synchro-generator which transfers its angular position to an indicating synchro-motor during the indication period in a manner similar to that of the minute volume recorder. The stepping switch is then reset to its zero position.

Safety Features

The common use of explosive gas mixtures for anesthesia makes it necessary that potential ignition sources be supplied with safeguards to

prevent the occurrence or propagation of explosions. Electrical equipment of the type required in the operating room console contains voltage sources, contactors, motors, and numerous hot filaments, all of which could serve as sources of ignition under certain conditions. Equally important as ignition sources are possible static charge accumulation on the surfaces of the equipment and the accidental connection of the instrument housing or any of its parts to the power line through insulation failure.

To prevent the accumulation of electrostatic charges, the operating room console is equipped with conductive rubber casters grounded to the metal frame. However, the use of explosion-proof fixtures, as in fixed electrical installations for power line equipment, did not appear feasible for portable equipment of this size. Instead a system based on the maintenance of a small positive pressure within the enclosure was designed. This made it unnecessary to seal the enclosure hermetically, although reasonable care was taken to keep the leakage small by restricting the size of openings. Thus, for example, the instrument panel is covered with a sheet of safety glass and sealed to the instrument case by means of a rubber gasket and compression frame.

A sliding vane compressor in the recorder console maintains the interior of the operating room console at about one-half pound pressure above that of the outside. Noise is eliminated by an acoustic filter located in the inlet side of the compressor. The output air of the compressor is first cooled by passage through a heat exchanger before it is delivered to the operating room console. Safeguards are provided which prevent the application of any power to the unit in the operating room unless the purging pressure is present. Also, a time delay inserted in the power line insures that the purging system must be in operation for about two minute before power can be applied to the unit.

To insure the safety of the patient against possible short circuits or other accidental sources of electrical hazard, the electrodes used in measuring heart beat are designed so that high-ohm resistors can be placed in each of them. Safety provisions have also been incorporated in the blood pressure measurement device to guard against the application of excessive pressure to the arm of the patient or, what is worse, the maintenance of an occluding pressure for long periods. Excessive pressures are prevented by a mechanical pop-off valve which opens and relieves arm-band pressure when the system pressure exceeds a pre-set value. In addition, an adjustable electrical limit switch is set to cause deflation of the arm band whenever the system pressure exceeds a value reasonably well above the patient's systolic pressure. In the event of power failure, air is discharged from the system by a solenoid-operated exhaust valve which normally functions to discharge the system at the end of each blood-pressure determination.

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SWANN, W. F. G.: Acceleration to Cosmic Ray Energies by Betatron Action.
BARLOW, JOHN S., GEORGE W. FREY, JR. AND JOHN B. NEWMAN: Very Low Frequency Noise Power from the Lightning Discharge.
WRIGHT, FRANK LLOYD: American Architecture.
LEVITT, I. M.: The Hydrogen Bomb.
SEYMOUR, D. W. AND W. F. G. SWANN: A Determination of u-Mesotron Rest Lifetime and a Verification of the Relativistic Expression for Variation of Lifetime with Momentum.
TEA, PETER L.: A String Net Analog for the Numerical Solution of the Equation of Laplace and of Poisson.
SCHONLAND, B. F. J. AND D. J. MALAN: Upward Stepped Leaders from the Empire State Building.
SCARLOTT, C. A.: Wind Tunnels—Why and Where.

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NOTES FROM THE FRANKLIN INSTITUTE MUSEUM

A. C. CARLTON, DIRECTOR

PERPETUAL MOTION

The search for perpetual motion never seems to die. It has been the means for promoting some impudent frauds upon the public, and one of the most interesting of these was perpetrated by Charles Redheffer of Philadelphia, almost 150 years ago.

Redheffer, with the ardent support of the newspaper *Aurora* and some influential people who should have known better, built and operated his machine in a building on the bank of the River Schuylkill. The mechanism, such as it was, deceived so many people who had paid a dollar a head to see it in operation, that the inventor was emboldened to offer it in return for a sizable financial consideration, to the Commonwealth of Pennsylvania who were to gain in prestige by the ownership of such a prize. Before accepting the offer, the Legislature proposed to investigate the inventor's claims. A committee was appointed consisting of such men as Oliver Evans, Henry Voight, Nathan Sellers, and Robert Patterson, who were to examine the machine and to pronounce upon its merits.

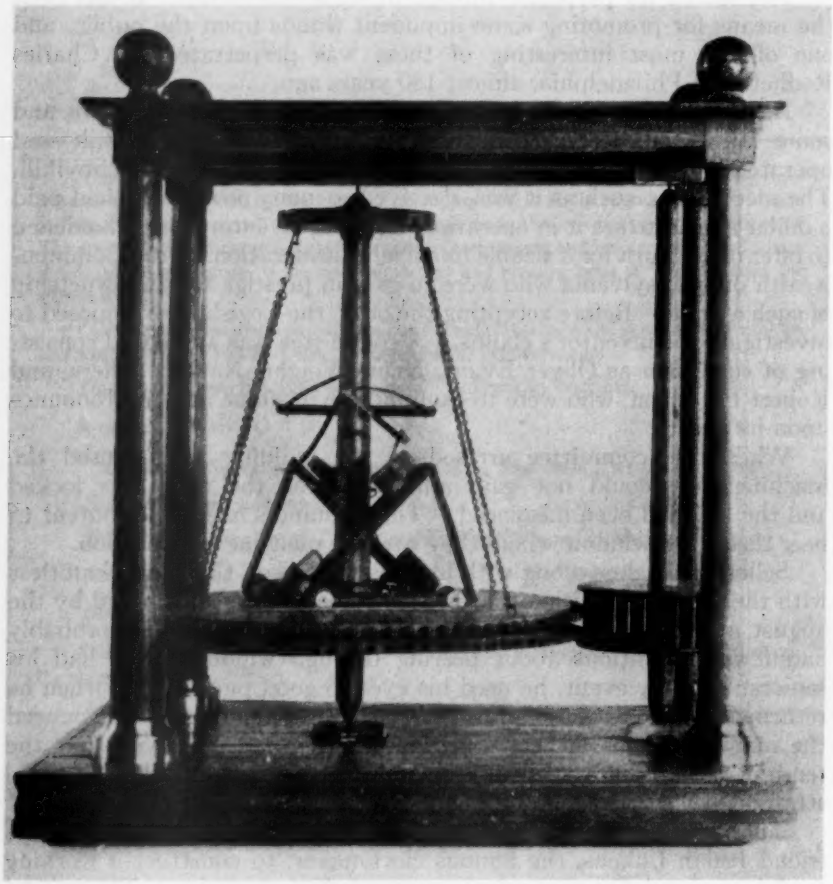
When this committee arrived at the building that housed the machine they could not gain admittance as the door was locked and the key had been misplaced. The examiners had to be content to peer through a window where they saw the machine in operation.

Sellers had taken along with him his young son Coleman, doubtless with the stern admonition that he was to be seen but not heard by the august members of the committee. Young Coleman Sellers probably had fewer inhibitions about peering through windows than had his seniors. In any event, he used his eyes to good purpose for, when he returned home and the restraint of silence was removed he denounced the machine to his father as a swindle. He had perceived that the teeth of the driving wheel were worn on the wrong face, which demonstrated that, so far from doing any driving, it was being driven.

The outcome of this revelation was that Sellers senior asked his friend Isaiah Lukens, the famous clockmaker, to construct a working model operated by a cunningly concealed clock-spring. This model when completed was displayed to a number of friends and, eventually, to the inventor himself. Redheffer was so dumbfounded that he offered Sellers a substantial share in his rights to the machine in return for the secret of its operation !

This episode finished Redheffer's pretentions locally, and he went to New York where his success was repeated (at a dollar a head) until

Robert Fulton visited the premises, and became so incensed that he kicked a hole in the plaster wall and exposed an old negro turning a crank that gave the machine its motion. On taking the machine to England after this second exposure, Redheffer was so bitterly attacked by the press over his claims that he did not venture to exhibit the machine as he had intended.



The original Lukens model that exposed Redheffer's claim to have found perpetual motion.

All that remains of the great fraud is the original model by Lukens, which is exhibited in the Museum. Visitors are recommended to examine it carefully to see whether they can detect the hidden source of power before asking for an explanation.

NOTES FROM THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

NICOL H. SMITH, DIRECTOR

PHYSICAL PROPERTIES OF TOBACCO

BY

RAYMOND AMICONE

The general public has recently become familiar with various research programs that include the study of the physiological effects of cigarette smoking. Much less familiar, except to people in the industry, are the intensive investigations that have been made on the growing, curing and smoking qualities of tobacco. Few industries can rival the energetic, widespread research programs conducted by the tobacco industry.

Yet as widespread as this research is, few studies have been made of the basic mechanical properties of tobacco. The behavior of tobacco under tensile, compressive, and shearing forces and the effects of time, temperature, and moisture are largely conjecture. Even in the manufacture of cigarettes, where mechanical genius has produced machinery capable of making more than 1200 cigarettes per minute, rule of thumb methods generally prevail.

To improve the design of the machines which manipulate and shear tobacco at high velocities, a better understanding of the mechanical behavior of tobacco itself is necessary. Hence the American Machine and Foundry Company, manufacturers of cigarette machinery, have begun a study of this behavior. Part of the program is now being carried forward by the Engineering Physics Section of The Franklin Institute.

Because of the possibilities for immediate application, studies at the Institute where begun on the measurement of stress relaxation in bulk tobacco constrained to definite volumes. These measurements are made possible through the use of a small pressure-sensitive capacitative device developed at these laboratories. This pressure element is buried in the center of a mass of tobacco that is rapidly compressed to a smaller volume. While the new volume is held constant, the pressure-sensing device measures the stress in the tobacco. This measurement gives the pressure, or stress, as a function of time. The data follow the general form:

$$S = C/t^n$$

where

S = stress
t = time
C and n = constants.

These preliminary data also indicate that moisture content may play a significant role in the determination of the relaxation constant (n). This is probably no surprise to cigarette manufacturers, who are continually plagued with either lack or excess of moisture in every stage from the growing of the leaf to the packaging of the cigarettes.

The ambitious program outlined by American Machine and Foundry is directed toward an understanding of the fundamental physical properties of tobacco and represents a major attempt to substitute sound principles for trial and error methods in the development of better cigarette machinery.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

ELLICE McDONALD, DIRECTOR

2-DEOXY-D-GLUCOSE AS AN INHIBITOR OF CANCEROUS GROWTH IN ANIMALS

BY

J. O. ELY

The possibility that 2-deoxy-D-glucose (2DG) might inhibit tumor growth *in vivo* was indicated from *in vitro* studies with 2DG previously carried out at this laboratory (1–5). 2DG had been shown to act as an antimetabolite of glucose in fermentation and growth of yeast, in growth of embryonic chicken heart cells in tissue culture, and in glycolysis of tumor and other rat tissues. Administration of 2DG to mice has now been shown to inhibit to a considerable extent the growth of the Krebs ascites carcinoma, to inhibit to a lesser extent the growth of two subcutaneously grafted tumors, but to have no retarding effect on the course of a myelogenic leukemia.

EFFECT ON KREBS ASCITES CARCINOMA

White mice of the Detwiler strain, weighing approximately 20 g. each, were used. One hundred and sixty-one mice were inoculated intraperitoneally with different amounts, as indicated in Table I, of the ascitic fluid from mice bearing the carcinoma. Treatment with 2DG was begun 24 hours later. A 5 per cent aqueous (isotonic) solution was given subcutaneously and in doses as indicated in Table I.

TABLE I.—*Effect of 2DG on the Survival Time of Mice Bearing the Krebs Ascites Carcinoma.*

Expt. No.	Inoculum Ascitic Fluid, ml.	No. of Mice	2DG Treatment doses/day	2DG Treatment mg./dose	Average Survival Time Treated, days	Average Survival Time Controls, days	Increase in Survival Time of Treated Mice, %
1a	0.1	25	1	10	6.5	5.5	18.2
1b	0.1	25	2	5	6.5	5.5	18.2
2	0.025	46	1	10	11.9	9.3	28.0
3	0.025	15	3	10	12.3	11.3	9.0
4	0.005	30	1 or 2	10	13.0	8.9	46.1
5	0.005	20	1 or 2	10	14.0	9.4	48.9
6	0.005	15	1	10	16.3	13.4	21.6

One hundred and sixty-one control mice in some groups received 0.85 per cent NaCl in volume corresponding to the 2DG dosage, and in other groups received no treatment. There was no difference in survival time of the NaCl-treated or untreated control mice.

The results (Table I) show that the survival time was increased in all the groups receiving 10 or 20 mg. of 2DG daily regardless of conditions of treatment. In groups 1a and 1b, both of which had received the largest inoculum (0.1 ml.), the survival time was increased about 18 per cent whether two 5-mg. doses or one 10-mg. dose was given daily. In group 2, where the amount of inoculum was cut to 0.025 ml., the survival time was increased to 28 per cent by the same dose of 2DG as in the preceding group. Thirty mg. of 2DG given in 3 doses of 10 mg. daily proved harmful to the mice: absorption between doses at 8-hr. intervals was incomplete and the subcutaneous tissue became edematous. The survival time was increased only 9 per cent. Best results were obtained when 10 mg. of 2DG were given alternately once and twice daily (20 doses in 13 days) to mice which had received a small inoculum (0.005 ml.); in 2 separate groups, the survival time was increased 46 and 49 per cent.

EFFECT ON SOLID TUMORS AND ON LEUKEMIA

Each of the treated mice in the following groups received subcutaneously 10 mg. of 2DG (0.2 ml. of 5 per cent solution) once daily. Size of the solid tumors was calculated as volume from measurements of length and width of the tumors.

2DG treatment of C57 Black mice bearing Bladder Carcinoma 150 was begun 6 days after tumor implantation. After 12 days of treatment, the mean tumor size of 24 mice was 92 per cent as large as that of 24 untreated mice. The average survival time of the treated group was 7 per cent longer than that of the untreated group.

2DG treatment of 15 Webster mice implanted subcutaneously with Carcinoma 687 was begun 24 hours after tumor implantation. After 20 days of treatment, the mean tumor size was only 81 per cent as large as in a control group.

No beneficial effect of 2DG was observed in C57 Black mice with myelogenic leukemia C1498. Treatment was begun 24 hours after inoculation with the leukemia. The survival time of 25 treated mice and of 25 control mice was in each case 12.7 days.

EFFECT ON GROWTH OF IMMATURE MICE

Normal immature mice of the Detwiler strain, 15–17 g. in weight, were treated subcutaneously with 10 mg. of 2DG once daily. After 16 days of treatment, as also after 23 days of treatment, the mean increase in weight of 25 treated mice was only 56 per cent as much as that of 25 control mice.

FATE OF INJECTED 2DG

Paper chromatographic methods were used for detection of 2DG in tissue homogenates, blood and ascitic fluid of mice and in urine of

rats after 2DG treatment of the animals. The method is sensitive to 1–2 μg. of 2DG.

2DG and also glucose were found in rat urine 1–7 hours after intraperitoneal injection of 100 mg. of 2DG into rats of approximately 250 g. in weight. Since injection of an equal amount of glucose did not cause the appearance of glucose in the urine, the 2DG appears to have an effect on the carbohydrate metabolism of the animal.

2DG was found in blood, liver, kidney, intestine, and at the site of injection 10 minutes to 6 hours after subcutaneous injection of 25 mg. of 2DG into mice. After subcutaneous injection, a large part of the 2DG is sequestered in the subcutaneous tissue into a gelatinous sac from which it is gradually absorbed.

2DG could not be found in homogenates of solid Krebs carcinoma or Carcinoma 687 when tested at various intervals between 15 minutes and 6 hours after subcutaneous injection of 20 mg. of 2DG into mice bearing these solid tumors. However, 2DG was found (Table II)

TABLE II.—*Glucose and 2DG in the Ascitic Fluid of Mice Bearing the Krebs Ascitic Carcinoma.*

Time Elapsed after Injection of 2DG, min.	2DG Present	Glucose Present
15	−	−
30	+	−
45	++	+
60	+++	++

in the ascitic fluid from mice with the Krebs ascites tumor after subcutaneous injection of 20 mg. of 2DG. The amount present increased between 15 and 60 minutes after injection. The appearance of 2DG was followed by the appearance of glucose in the ascitic fluid, the amount of glucose increasing in proportion to the amount of 2DG. The absence of glucose in the ascitic fluid initially is probably due to complete utilization by the tumor cells as it appears in the fluid, since tumor cells are known to convert glucose rapidly to lactic acid. The accumulation of glucose then is probably due to 2DG inhibition of the glycolytic utilization of the glucose, since *in vitro* experiments (3) had shown that the glycolysis of tumor cells was inhibited by 2DG.

EFFECT OF 2DG ON LIVER GLYCOGEN CONTENT

Since 2DG caused the appearance of glucose in the urine, the effect on liver glycogen content was investigated. 2DG in 5 per cent aqueous solution was injected into rats of the Wistar strain which weighed about 150 g. The rats were then fasted for various time intervals after which liver glycogen determinations were made. Glycogen depletion of the

TABLE III.—*Effect of 2DG on Liver Glycogen Depletion.*

No. of Rats	2DG Injected, mg.	Period Fasted, hr.	Liver Glycogen Content Treated, %	Controls, %
4	315	5	4.7	3.4
2	315	7	1.6	0.15
5	315	17	2.1	0.6
5	50	17	0.7	0.1

liver caused by fasting was retarded but not prevented by the 2DG treatment (Table III).

Re-glycogenation of the liver was not affected in a 160 g. rat which had been fasted 24 hours before 2DG treatment. When 100 mg. of 2DG was injected subcutaneously and the rat refed, liver glycogen was 2.7 per cent after 7 hours of refeeding; the liver glycogen of a control rat was 2.9 per cent.

CONCLUSIONS

The survival time of mice with Krebs ascites carcinoma was increased by the administration of 2DG. The increased survival time probably was due to an indirect effect and also to a direct inhibitory effect. The indirect effect can be assumed since 2DG treatment retarded the growth of immature mice. 2DG could not be found in solid tumors, and since 2DG treatment retarded the growth of immature mice, the retardation of growth of the solid tumors may be a reflection of the impaired carbohydrate metabolism of the animal. The direct inhibitory effect on Krebs ascites carcinoma is apparent because 2DG was found in the ascitic fluid and because there was an accumulation of glucose in the fluid which indicated that the glycolysis of the tumor cells was inhibited by 2DG.

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BOOK REVIEWS

COPPER. THE METAL, ITS ALLOYS AND COMPONENTS, edited by Allison Butts. A.C.S. Monograph No. 122. 936 pages, diagrams, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $20.00.

This text is another in a long series initiated by the American Chemical Society in 1921. It is well to keep in mind the intended principal purposes of this particular monograph series. These are restated in the introduction of the book as follows: "first, to make available to chemists a thorough treatment of a selected area in form usable by persons working in more or less unrelated fields to the end that they may correlate their own work with a larger area of physical science discipline; second, to stimulate further research in the specific field treated. To implement this progress the authors of the monographs are expected to give extended references to the literature. When the literature is of such volume that a complete bibliography is impracticable, the authors are expected to append a list of references critically selected on the basis of their relative importance and significance."

In view of these purposes, the reader of an A.C.S. monograph would expect a thorough, up-to-date, well-referenced text on a particular subject. It is doubtful if the present book would fulfill such expectations; however, it does contain a wealth of information and will undoubtedly prove of considerable value as a reference text. In using it as such, the reader will have to keep in mind that not all of the subjects treated have complete bibliographies or present data recent enough to be in keeping with the publication date of the book.

For example, in the chapter on the physical properties of copper, the tensile properties at low temperatures are summarized in a graph taken from the work of C. A. Smith in 1939 and Colbeck and MacGillivray in 1933. In this case, it would have been better to have used, or at least to have included, the excellent data on the same subject obtained by Geil and Carwile at the National Bureau of Standards in 1951. More recent information on this subject has been published by Carreker and Hibbard (1953); but these more recent data were probably not available to the author.

The roasting, smelting, and converting of copper are satisfactorily covered in forty-seven pages; however, it would have added to the completeness of this section to have referenced the well-known 1942 text, "Metallurgy of Copper," by Newton and Wilson which covers this same material in much greater detail.

This same criticism can not be applied to all of the monograph. An excellent chapter that adds greatly to the value of the book is that written by Prof. R. M. Brick on the phase diagrams of copper. This section constitutes a thorough review of the binary phase diagrams and some of the ternary phase diagrams up to 1953. These are not to be found in any other single text and thus provide one with a convenient, dependable reference on the subject. Such a treatment fulfills the purposes of the book as set forth in the introduction.

Professor D. W. Wakeman's chapter on the physical chemistry of copper is in sharp contrast to this thorough review. For instance, this author presents a table on the diffusion of various elements in copper. This table is taken from the "Metals Reference Book" by Smithells. Such a table could be of considerable value if it were complete and up-to-date. Unfortunately, Smithells' data on the diffusion of various elements into copper were not only incomplete when they were published in 1949 but also were not particularly well chosen. For example, the value of the diffusion coefficient, D_o, of 47.0 $cm^2/sec.$ for copper self-diffusion as determined by B. V. Rollin, is unusually high. More realistic values are those of investigators not cited. This chapter might better have been entitled the Physical Metallurgy of Copper since it deals with the following subjects: diffusion, alloying behavior, intermediate phases in copper sys-

tems, mode of deformation, ternary equilibrium and order-disorder.

To cite several poor chapters and only one good chapter would be inaccurate. There are many sections that are complete, well written and satisfactorily referenced. Some of these are: Thermodynamics of Copper Smelting, Electrolytic Copper Refining, The Hydrometalluurgy of Copper, Wrought Copper Alloys, and Hot Working of Copper and Copper Alloys. The latter chapter is particularly good. It is written with sufficient detail to make it extremely useful to any one interested in the hot working of copper. Excellent illustrations of hot working equipment add considerable clarity to the text.

The editor has allowed the following approximate percentages of space in the monograph; 39 per cent, occurrence and extractive metallurgy; 16 per cent, physical and mechanical metallurgy; 18 per cent, properties and application; and 27 per cent, various aspects of the chemistry and physics of copper. There are forty-six chapters ranging in size from seven to fifty-eight pages per chapter. Six chapters have no references and two of the chapters combined have 260. There are a total of 1320 references.

Individual chapters are devoted to specialized topics, such as copper and copper alloys in the electrical industry, copper in the building industry, automotive applications of copper and copper alloys, the use of copper and copper alloys in ordnance, copper in biology, copper as an agricultural fungicide, etc. The longest chapter in the book is that on cathode melting and casting of marketed copper shapes, which in the opinion of this reviewer is overweighted.

In general this book will prove to be a useful reference text. However, one can not help but feel that the diversification has been made so great so as to preclude adequate coverage for most topics.

R. L. SMITH

ASTROPHYSICS. THE ATMOSPHERES OF THE SUN AND STARS, by Lawrence H. Aller. 412 pages, diagrams, 16 × 24 cm. New York, The Ronald Press Co., 1953. Price, $12.00.

In 1946, this reviewer took a graduate course in astrophysics and for a text he used a typewritten copy of a book on astrophysics which was written by Dr. Lawrence H. Aller. Now, seven years later the typewritten text has given way to a formal book. The point made is that as long ago as 1946 there was needed an astrophysics textbook on the graduate level and finally it has materialized.

Dr. Aller, who is associate professor of astronomy at the University of Michigan, has written a singularly competent work on a highly technical facet of astronomy. This work involves the atmospheres of the sun and stars.

Not only are the major developments in this field delineated but the fundamentals necessary for the understanding given. The author states in the preface: "Throughout this book I have stressed not merely the results but also the methods by which they are obtained. The reader is shown how each principle or important formula is applied to some definite numerical problem concerned with the interpretation of the stars and nebulae." Herein lies one of the significantly favorable aspects of this work. Textbooks should guide the student and this approach is designed to furnish that guidance.

It cannot be indicated too strongly that the astronomer is seriously handicapped in his investigations. Whereas other scientists can use five senses, the astronomer has only one—the visual sense. It is with this single sense that he attempts to interpret natural phenomena in the sun and stars. And if his results appear inconclusive and uncertain there is good reason for these shortcomings. Throughout this work this uncertainty and these apparent contradictions are apparent. These do not detract from the value of the book because the researches which gave rise to the results are basic.

Astrophysics is divided into nine chapters. The first gives the basic astronomical data and the physical background necessary for interpretation of stellar bodies. Chapter 2 deals with the salient facts of spectroscopy. Chapter 3 reviews the gas laws. This proceeds from the perfect gas to the degenerate gases and concludes with the turbulence of large-scale mass motions of gases. Chapter 4 deals with the ionization and dissociation equations. Chapter 5 discusses the radiation

laws, transition probabilities and the absorption and emission of radiation.

The first five chapters lay the foundation for the remaining four chapters.

Now the intricate problem of analyzing the atmospheres of the sun and stars from their spectra is undertaken. The results are eminently commendable.

Chapter 6 deals with the radiation of the stars while in Chapter 7 the continuous spectra of the sun and stars are detailed. Chapter 8 deals with absorption spectra. The author states: "The fundamental problem of stellar atmospheres is the following: Given the intensities of the absorption lines, determine the composition and physical state of the atmosphere and the surface gravity of the star. From the spectrum we should be able to tell whether the star has a normal or an unusual composition and whether it is a dwarf, giant or supergiant." The last chapter relates our knowledge of solar phenomena.

Dr. Aller has written a well balanced, comprehensive book on a subject which is in a state of flux. So much is being done on astrophysical problems that portions of this book will be obsolete soon. However, the fundamentals are so firmly integrated that this work will have significant distribution, even in the future.

The problems and bibliographies at the conclusion of the chapters enhance this work. Dr. Aller is to be congratulated on fulfilling a specialized need in astronomy.

I. M. LEVITT

ELECTRONICS, by George F. Corcoran and Henry W. Price. 459 pages, diagrams, 16 × 24 cm, New York, John Wiley & Sons, Inc., 1954. Price, $7.00.

Although the subject popularly referred to as Electronics is extremely broad in specific applications, the fundamental concepts necessary for understanding its basic principles are not so numerous as might be thought. As one examines electronic systems embodied in communications devices, radar, computers or industrial controls, it becomes plainly evident that they are composed of common electronic building blocks which by definition must contain electron tubes or devices such as transistors. Hence, by avoiding discussions of these systems and confining their attention to the electronic building blocks, the authors of a volume entitled simply "Electronics" have been able to limit the extent of their book to manageable proportions.

Where circuit applications utilizing certain vacuum tube characteristics are mentioned, the reference is brief and is included for illustrative purposes only. The bulk of the descriptive and mathematical presentation pertains directly to the underlying electronic behavior responsible for the characteristic discussed. Thus, the reader is introduced to the fundamental mathematical relations and conceptual knowledge required to design circuits.

As a convenient reference for vacuum tube circuit mathematics, this book can be useful even to the experienced engineer. In addition, circuit relations governing the operation of transistor networks are provided to the extent that an engineer inexperienced in the use of the new element should be able to design circuits comparable to former vacuum tube applications. The authors also make a point of showing triode-transistor analogies in the various forms of circuits as well as a comparison of the mathematical expressions governing them.

Excluding the material on transistors and transistor circuits, this volume could have been written years ago. However, it is not easy to prepare a cohesive course on electronics, avoiding the temptation to stray into discussions of system applications or overemphasize the field in which the author's interest and experience are strong. The authors of this short text have apparently regarded their material critically, retaining only that which bears on electronics as an engineering fundamental. As an illustration, one may note that there is no treatment of tuned circuits in spite of their intimate association with the tube of an amplifier or oscillator. This omission is justifiable, as are the other deviations from conventional contents; for, when an author becomes committed to a treatment of tuned circuits, he may feel obliged to cover the subject of radio engineering before concluding. The contents have therefore been selected according to the logical implication of the term

"electronics" rather than according to text book convention.

What has been written above would give a mistaken impression if it appears that Corcoran's and Price's book is written in a popular or elementary tone. The material is treated at an engineering level, and use of the book would be justifiable in a college engineering course. Many engineering schools make a practice of giving a brief course in electronics fundamentals to engineers not majoring in this field because a large variety of practical instrumentation is designed around vacuum tube building blocks. Students of engineering in general possess the mathematical training to make use of this book which should permit them to design much of the electronic apparatus which sooner or later may confront and perhaps confound them.

C. W. HARGENS

PHYSICAL METEOROLOGY, by John C. Johnson. 393 pages, diagrams, 16 × 24 cm. New York, John Wiley & Sons, Inc.; Cambridge, The Technology Press; 1954. Price, $7.50.

The title of the book is a misnomer, for instead of it being *Physical Meteorology*, it should have been titled "Physics of the Atmosphere." This in no way detracts from the value of this work for it is a significant contribution from many disciplines such as astronomy, chemistry, radio physics, physics of the air and meteorology carefully integrated into a coherent, comprehensive picture of what takes place in the atmosphere.

This is not a book which can be read. It is designed primarily as a book to meet the needs of researchers and students at the college level. Its use presupposes a thorough knowledge of physics plus complete familiarity with differential and integral calculus. *Physical Meteorology* as such is an important contribution for it fills a distinct need as a textbook.

The book is divided into eleven chapters of which the first three involve atmospheric refraction, scattering in the atmosphere and the theory of atmospheric visibility. The next two chapters deal with radiation processes in the earth's atmosphere and radiation transfers in and out of the earth which tie the previous chapters together as parts of the complete picture.

Chapter 6 involves atmospheric optics with complete description of halves, coronas, rainbows and other atmospheric phenomena. This is one of the most eloquent treatments this reviewer has seen on these visual effects.

Chapter 7 deals with the conditions attending the formation of cloud particles while the next chapter deals with an entirely new subject not ordinarily found in books—cloud seeding. This treatment is all too brief but the uncertainties in this field may have been the reason for Dr. Johnson hedging on this topic.

Chapter 9 deals with atmospheric electricity in the classical fashion while Chapter 10 takes up the problem of the ionosphere and ozonosphere.

The last chapter deals with the physical characteristics of the upper atmosphere.

There are some omissions and loose statements to which this reviewer, as an astronomer, is sensitive. In the section on aurorae no mention is made of the highly significant work of Dr. Meinel in determining the radiation responsible for these lights. In the section on the sun the mean solar distance should be 93×10^6 miles. The word "molten" on page 103 should not be used. At the temperatures of the sun, it is completely gaseous.

These are minor errors of no real consequence and do not in the least detract from a highly significant book.

I. M. LEVITT

INTRODUCTION TO NUCLEAR ENGINEERING, by Richard Stephenson. 387 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $8.00.

It is interesting to note that this book comes less than 20 years after the discovery of fission. It was only the intense war effort that made such progress in nuclear reactors and nuclear engineering possible. Nuclear power is no longer a dream but is a reality now.

This book is intended for engineering students and so presents the fundamentals of nuclear physics and fission in two introductory chapters. Then follows material on

reactors and reactor theory, radiation shielding, and materials of construction. Finally come chapters on instrumentation and control, separation of isotopes, chemical separations and processing, and special techniques. There are appendices on constants, radioisotopes, neutron absorption cross-sections, and properties of moderators. References are given to important source material.

The author has turned out a first-rate text. The material is written in good style, it is nicely divided by headings and subheadings, examples are used to help explanations, problems of a practical nature are given at the end of every chapter, and the illustrations and graphs are excellent. One of the chief merits of the book is that explanations are based on first principles as much as possible. The fact that it is written as a text in no way impairs its usefulness as a reference in the field of nuclear engineering.

Emphasis seems to have been placed on the engineering aspects of operation of reactors rather than on applications. For this reason, practically no mention is made of nuclear power. This is somewhat disappointing.

It is a pleasure to see a new book of considerable merit in the new field of nuclear engineering. LEONARD MULDAWER

ADVANCED ORGANIC CHEMISTRY, by E. Earl Royals. 948 pages, diagrams, 16 × 24 cm. New York, Prentice-Hall, Inc., 1954. Price, $10.00.

This treatise is intended for use in a first-year graduate course or in any second course in organic chemistry following a thorough elementary course. The emphasis is on "transformations of organic matter, presented in such a manner as to reveal fundamental behavior patterns in terms of the unifying principles of electron theory." Primary attention is paid to the essential basic principles. The book is divided into two sections. The first part describes the structure and reactivity of organic compounds, the alkanes, cycloalkanes, alkenes, alkynes, and aromatic hydrocarbons. The second part is devoted to the nature of the carbonyl group, methods of preparation of carbonyl compounds, their addition reactions, and their reactions dependent upon active hydrogen. Copious references are given to the original literature. The index to authors cited covers eleven pages of three columns each; the subject index covers ninety pages of two columns each.

JOSEPH S. HEPBURN

TIME COUNTS, by Harold Watkins. 274 pages, plate, 15 × 22 cm. New York, Philosophical Library, 1954. Price, $4.75.

One of the most fascinating chapters in our civilization delineates the growth of our calendar. It is woven into the warp and woof of our mental fabric and any attempt to show its evolution has, of necessity, a historical background.

It was the Earl of Chesterfield who coupled chronology with geography. He called these "the two eyes of History, because History can never be clear and understood without them. History relates facts; chronology tells us at what time they were done; and geography shows us in what place and country they were done." Because of this historical background the calendar has aroused and maintained its interest to man.

In *Time Counts,* Harold Watkins relates the story of the calendar; its beginnings, its present state and the future trends. The book is divided into three parts in which the first deals with the history of the calendar from its origin to the present day. Tucked away in these 80 pages is a wealth of solid fact dealing with the calendars of antiquity. Also here is discovered a refutation of the oft-quoted remarks about the riots in London and Bristol associated with the English reform of the calendar. This reviewer has made this statement many times. Here, for the first time, he finds a researcher who has investigated the original for these acts and comes up with a negative answer. Mr. Watkins is to be congratulated on laying this ghost to rest.

The second part deals with contemporary calendar reforms. After the Gregorian Calendar introduction it was soon discovered that there was still a long term error in the calendar and what was more important, our modern civilization had completely out-

stripped the calendar and had made subterfuges necessary in order for reasonable records to be kept. This section deals with these attempted reforms. Here is also contained a tribute to that magnificent lady and founder of the World Calendar Association, Miss Elizabeth Achelis. If the new World Calendar is ever introduced in our time it will be due directly to her untiring efforts.

In part three are described calendrical activities in various groups and nations. It is really a plea for adoption of a universal calendar for all the world to end the individual chaos which exists.

Time Counts is a fascinating story of the calendar. It should be read by all to shed the apathy surrounding the efforts to reform the calendar.

I. M. LEVITT

PRINCIPLES OF NUMERICAL ANALYSIS, by Alston S. Householder. 274 pages, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1953. Price, $6.00.

With the growing importance and increasing availability of high-speed digital computers, there is a need for books such as this one. This book, however, is of value to those who do hand computation as well as to those who devise computation routines for high-speed computers. It is not written like a cook book, but rather as a textbook with clear presentations of the principles underlying computation routines.

The following is a summary of the topics discussed in the book: Chapter 1 is concerned with various types of errors which occur in computation procedures, their propagation, and how to evaluate them. Chapter 2 deals with matrices and linear equations. This chapter opens with a detailed discussion of vectors, matrices, determinants and their properties. This is followed by consideration of various iterative methods in linear systems and the error analysis of iterative procedures. Direct methods in linear systems are then described, and finally comparison of the different procedures is made on the basis of the number of operations involved in each method.

Chapter 3 is on nonlinear equations and systems. It opens with a discussion of zeros of polynomials, their properties, and how their locations are determined. This is followed by a discussion of Graeffe's root squaring method for determining zeros. Several other methods are discussed. These methods do not converge as rapidly as Graeffe's; however, they have the advantage that errors tend to die out rather than propagate themselves as they do in the Graeffe process. This chapter also contains a brief account of methods for dealing with systems of nonlinear equations.

Chapter 4 is concerned with proper values and proper vectors of a matrix. Methods for obtaining these are divided into two distinct classes: iterative methods which manipulate the matrix, and direct methods that deal explicitly with the characteristic function of the matrix to obtain a proper vector from its corresponding proper value.

Chapter 5 considers the problem of interpolation, that is, determining a function that has given values, and for values of its derivatives at specified points. Polynomial interpolation is described in detail, trigonometric and exponential interpolation is considered briefly. Chapter 6 deals with some extensions of the methods of interpolation, including least-square curve fitting. Chapter 7 is devoted to methods of numerical integration and differentiation.

Chapter 8 contains a brief exposition of the Monte-Carlo method. It is illustrated by a random sampling scheme for obtaining the integral of a function of several variables.

The concluding section of each chapter is devoted to a description of the bibliographical material on the subject of the chapter. This will be of great assistance to those who will want to delve more deeply into special topics.

H. L. PLATZER

THE MECHANICS OF ENGINEERING SOILS, by P. Leonard Capper and W. Fisher Cassie. Second edition, 315 pages, diagrams, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $6.50.

The first edition of *The Mechanics of Engineering Soils* appeared six years ago and at that time was one of the few books on the relatively new science of soil mechanics. Since then a number of books on the subject have been written. The second

edition of *The Mechanics of Engineering Soils* includes rewritten material on shearing resistance, bearing capacity, pavement design and flow nets, and a new section on soil suction. In general, while the book may not be as theoretical a text as Terzaghi's "Theoretical Soil Mechanics," it nevertheless contains all the basic material on soil mechanics and serves as a good introduction to the subject.

Until recent years, the building of retaining walls, various types of foundations, earth dams, roads and other civil engineering projects involving soils was essentially based upon empirical data. Although still relying to some extent upon experience, the science of soil mechanics—the application of the laws of mechanics and hydraulics to soils—has made rapid strides in the last two decades. Now, by making certain standard soil tests, the proper type of foundation for any superstructure can be determined accurately. The proper road beds for highways, the proper structure of earth dams and the proper means of stabilizing earth slopes are aspects of civil engineering which are pretty well known as a result of these advances in soil mechanics. All the various means by which the civil engineer solves the above problems are described in *The Mechanics of Engineering Soils.*

Following a description of the scope of soil mechanics, an historical survey and the classification of soils, the authors—P. L. Capper of the University of London and Professor W. F. Cassie of the University of Durham—describe various soil mechanics concepts. Moisture content, void ratio, permeability, compressibility and consolidation, shearing resistance, earth pressure, and stability of slopes and foundations are theoretically described and illustrated with practical applications. Settlement of foundations, pile foundations, design of roads and runways are described. Water seepage and its study by means and flow nets, means of controlling the watertable, and the drainage of roads and runways are discussed under drainage problems. The book concludes with methods of explorations for sites for proposed structures and means of sampling and testing of the surface and subsurface soils.

E. W. HAMMER

RELAXATION METHODS, by D. N. de G. Allen. 257 pages, diagrams, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1954. Price, $7.50.

Relaxation methods are coming into wider use every day. This book should spread the technique to a large group who have not previously used it. It is an excellent text for both self study and for classroom use. Any engineer who is unfamiliar with, or feels weak in relaxation methods could find no better text to use.

The main purpose of this text is to teach how to relax and the author does not stray from this objective. The student is taken step by step from the very elementary applications to highly complex problems. Each new idea is clearly stated and a worked example given. In fact, the book consists to a large part of worked examples illustrating the techniques introduced. Emphasis is placed on the processes which must become automatic to the engineer using them. The examples given in the text and the problems given at the end of chapters are drawn from the fields of electricity, magnetism, elasticity, mechanics and hydrodynamics. Several problems are given at the end of each chapter so that the student may apply the principles he has learned and get practice in the computational techniques. Practice is a necessity in learning this subject.

This book is not only a text for the beginner but includes recent advances in relaxation such as: three-dimensional relaxation; solution of the heat-conduction equation; the extension of the solution of other field equations to cases where the area of integration is not singly connected; and a new alternative solution for eigenvalue problems. Many subjects previously found only in journals can now be studied conveniently and with the proper background. Included are solutions of the biharmonic equation and of simultaneous differential equations, and a detailed description of the use of graded networks.

Four appendices, a bibliography containing some 87 references to journals including the author's contributions and a good index round out this text. It is a polished work with excellent detail in example and discussion. The clarity with which each topic

is presented makes this volume a pleasure to study. R. J. GIBSON

THE PRESENT STATE OF PHYSICS, arranged by Frederick S. Brackett. 265 pages, diagrams, 16 × 24 cm. Washington, American Association for the Advancement of Science, 1954. Price, $5.75 (to members); $6.75 (to non-members).

Physics has now become an all-embracing science encompassing more and more fields. It is becoming extremely difficult to differentiate these fields. As Dr. Brink puts it: "One way to characterize a field of science is to indicate the origin of the problems dealt with and the area of significance of the anticipated results." While biophysics is essentially biology the results are analyzed in physical terms. Now it is becoming an exact science so regimented that quantitative results are obtained. This is but one facet of the new physics.

In *The Present State of Physics* various disciplines are covered by acknowledged masters of their fields. It is divided into four sections with a total of ten papers which were presented at the meeting of the American Association for the Advancement of Science in New York at the end of 1949.

Since the symposium the authors have had the opportunity of augmenting their papers with significant data uncovered up to the present moment. This has at times been done by a footnote and at times in the body of the paper.

Since the advent of the large particle accelerators an entirely new field has been opened in the recognition of the elementary particles. Their nature, origin and interaction with matter have occupied the experimental and theoretical physicist during this time. The first part of this book deals with these particles. In all of this work the physicist is striving for a coherent picture involving the multiplicity of particles which are now recognized.

Transistor physics, ferromagnetics and the properties of semiconductors make up part two of the book.

Part three involves chemical physics dealing with the large molecules and the second paper is really an introduction to the last section which is on biophysics.

Part four comprises two papers, one by Brink and Johnson on nerve impulses and bioluminescence.

This volume reveals some of the most penetrating investigations in the above fields. I. M. LEVITT

HYDRAULIC SYSTEMS AND EQUIPMENT, by R. Hadekel. 221 pages, diagrams, 14 × 22 cm. New York, Cambridge University Press, 1954. Price, $0.00.

Hydraulic systems and equipment described by the author apply primarily to hydraulic mechanisms in which liquid is used to exert force and/or transmit and control motion. In general, the author covers hydraulic systems in which relatively incompressible liquids are used at fairly high pressures and in which fluid velocities are negligible from the standpoint of power transmitted.

The author has written a practical treatment of the subject which appears to be a valuable tool for the designer who has to work out hydraulic circuits which will perform specified functions in proper sequence and specified time. The book starts with a brief review of some of the principles of fluid mechanics, and some of the more basic concepts and configurations which are used in later sections of the book. Nomenclature and symbolic presentation of various valves, pistons and rotary machines are established. With these elements in mind, the author next considers groups of components or complete systems which are designed to perform specific operations. These systems are broken down into various categories and different types of controls are discussed.

Systems and circuits described can be applied to a variety of press operations and to injection machinery. Circuits are also described which could be applied to machine tool operations and remote control systems in general.

The author points out quite carefully matters of power dissipation and measures that can be taken to insure that standby or idling losses are minimized. He also discusses the matter of power economy and methods of energy storage that may be employed, when the duty cycle is such that there is an opportunity to store energy during idle periods.

The author includes a discussion of gen-

eral design considerations for hydraulic circuits, including such features as pipe sizes, working pressures, seals, filters and reservoirs.

The nomenclature and symbolic presentation of components is somewhat different from those we commonly use in this country. This difference is not sufficiently great, however, to impair the usefulness of the book. R. H. MARVIN

THE DETERMINATION OF CRYSTAL STRUCTURES, by H. Lipson and W. Cochran. Volume III in The Crystalline State, edited by Sir Lawrence Bragg. 345 pages, plates, diagrams, 15 × 23 cm. London, G. Bell and Sons, Ltd.; New York, The Macmillan Company; 1953. Price, $8.00.

The publication of this comprehensive and carefully prepared volume is an event of major importance, for the lack of a text on crystal-structure determination has long been felt. Many of the available texts on X-ray crystallography generally contain only summaries of the structure-determination techniques. Consequently workers and students in the field have found the material distributed piecemeal throughout the literature. Within this single volume it is possible to trace the path from the X-ray diffraction data to the final structure determination. However, it is not a pedantic description technique, but a lively discussion of the various methods liberally salted with examples from actual problems. This mode of presentation is particularly important for the student who may be reading the text as an initial introduction to the subject. The advanced worker, on the other hand, will not find the text over-burdened with minute details.

The book is of moderate length (341 pages) and is divided into nine chapters and an appendix. Each chapter concerns a particular phase and the chapters are logically arranged in the order in which a crystal-structure determination would proceed. The first chapter is an introduction to x-ray optics, and contains, as well as optics, a preview of the material that follows. The next three chapters form the groundwork, and are concerned with the use of space groups, calculation of structure factors, and the summation of Fourier series. Following these are four chapters which form the most important part of the whole book. These four chapters, entitled "Trial and Error Methods," "The Use of the Patterson Function," "Fourier Methods," and "Direct Methods," of course should form the nucleus for any discussion of crystal-structure determination. However, the appearance of such an extensive treatment is unique. Previously these important subjects were available in part in various texts and journals. Here for the first time, in a unified presentation, the authors painstakingly discuss the origin, use, interdependence and limitations of each method. They are to be sincerely complimented for a job well done. The closing chapter contains a discourse on the accuracy of the structure determination. Following this is an appendix which includes an excellent, up-to-date, bibliography.

In the preface of the book the authors make the statement that they have tried to cover the subject of crystal structure determination from the stage at which a set of structure amplitudes have been obtained to the final accurate positioning of the atoms. It can certainly be said that the authors have accomplished their goal.

ERIK KLOKHOLM

GEOMETRICAL MECHANICS AND DE BROGLIE WAVES, by J. L. Synge. 167 pages, diagrams, 15× 22 cm. New York, Cambridge University Press, 1954. Price, $4.75.

Sir William Rowan Hamilton developed a dynamical method for studying the motion of particles, and also an optical method for studying the paths of rays of light. Both methods are based on a variational principle. His dynamical method, which can be called "Newtonian geometrical mechanics," makes use of the principle of Maupertuis, and the "waves" in this theory are surfaces of constant action. The space involved here is ordinary three-dimensional space. Hamilton's optical method, which can be called simply geometrical optics, makes use of the Fermat principle, and the waves are fixed surfaces. The space again is three dimensional.

Professor Synge has posed the following question: Hamilton based his optical theory of rays and waves on a variational principle

in ordinary three-dimensional space, all three coordinates being treated on an equal footing; what sort of theory would one get by applying Hamilton's method to such a variational principle in four-dimensional space, x_1, x_2, x_3, x_4 (with $x_4 = ict$), treating all four coordinates on an equal footing? The result is a general and completely relativistic theory of de Broglie waves, which Synge calls "relativistic geometrical mechanics."

After a short Introduction (Chapter I) in which the general status and setting of the theory are described, the general theory of rays and waves in space-time is developed in Chapter II. The theory up to this point is strictly a "ray" theory, there being no "phase" involved; nevertheless, Synge shows that there is a fundamental distinction between ray velocity and wave-packet velocity, and that the connecting formula is identical in form with the familiar formula between phase velocity and group velocity.

Chapter III treats the geometrical mechanics of a particle, both free and in a given field, and here are introduced de Broglie waves. Chapter IV is on Primitive Quantization; the concepts of phase, frequency and wavelength are introduced, as in passing from geometrical to physical optics. The quantization is carried out by the introduction of a scalar wave equation (in four-space) which involves the Planck constant. It is then shown that the action along a ray between events (points x_1, x_2, x_3, x_4) of the same phase is precisely Planck's constant h. The theory is applied to the problem of a central force-field and the hydrogen-like atom, and to the Zeeman effect. Chapter V gives a generalization of the theory to N-dimensional space, and this generalization is used to discuss the two-body problem.

C. DOMENICALI

SOME BASIC PROBLEMS OF THE MATHEMATICAL THEORY OF ELASTICITY, by N. I. Muskhelishvili. Second edition, translated from the Russian by J. R. M. Radok, 704 pages, 16 × 24 cm. Groningen (Holland), P. Noordhoff Ltd., 1953. Price, fl 38.

The English translation of Muskhelishvili's Russian treatise on *Some Basic Problems of the Mathematical Theory of Elasticity* brings before students of elasticity a valuable addition to the present American and English literature on this subject. This translation was undertaken by J. R. M. Radok, while the publishers are P. Noordhoff Ltd., Groningen, Holland.

The book augments and parallels Love's "Theory of Elasticity" through additional contributions on two-dimensional elastostatic problems. This entire subject is presented in longhand form with little vector and tensor notation. An outstanding contribution is the presentation of methods of solution, as in the solution of series in the application of conformal mapping and the use of Cauchy integrals and boundary values of holomorphic functions in the application to the solution of special boundary problems in coplanar elasticity.

Chapters, 1, 2, and 3 cover the basic equations of elasticity. This notation follows that of earlier writers as in Love's "Theory of Elasticity." The second part of the book is on the General Formulae of the Plane Theory of Elasticity, covering complex representation, multi-valued displacements and thermal stresses with the solution of special problems. Part III covers several problems of the Plane Theory of Elasticity by means of power series, including conformal mapping. Parts IV and V are devoted to the application of Cauchy Integrals to the solution of boundary problems. Part VI extends to boundary problems by reducing to linear relations. Part VII covers such items as torsion and bending of bars, including different materials at different Poisson's ratios. The appendices are of interest. The first discusses the concept of a tensor, the second discusses multi-connected regions and the third covers functions of complex variables.

The book is a valuable contribution and its translation is well worth while for students in the field of elasticity.

RUPEN EKSERGIAN

INDUCTION AND DIELECTRIC HEATING, by J. Wesley Cable. 576 pages, illustrations, 16 × 24 cm. New York, Reinhold Publishing Corp., 1954. Price, $12.50.

This useful book presents in a quite descriptive manner the solutions to the many problems solved by industrial engineers

through induction and dielectric heating. Particular sections of the book are devoted to special classes of application. Sections of interest to metallurgists pertain to vacuum melting, heat treating, and forging. Of special appeal to the production engineer dealing in metal fabrication are the sections describing production set-ups for brazing, welding, soft soldering, and machine design.

Somewhat less spectacular, since lower temperatures are more common, is the less publicized field of dielectric heating. The technique of dielectric heating is shown to be especially invaluable in the plastic industry, the wood-working industry, the food processing industry and others.

A principal advantage of induction or dielectric heating is the ability to heat up a desired section of a specimen while remaining sections stay relatively cool. For example, the book describes set-ups for hardening by heat treating only the teeth of gears by induction heating while other sections remain relatively cool and therefore are not hardened. This localized heating characteristic of dielectric heating is advantageous in the processing of popcorn. The moist insides of a kernel heat more quickly than the skin in a high frequency field so that, using dielectric heat, it is no longer necessary to run the risk of scorching the skin to pop the kernel as was the case when direct heat was used.

The above mentioned applications typify the many detailed descriptions which make up the book. Very little of the book is devoted to the theory of induction and dielectric heating or to the development of generators, etc. The book is therefore more useful to users of high frequency heating than to designers or builders of high frequency heating equipment. However, the food processor or manufacturing engineer can obtain a none too detailed understanding of mechanisms of the equipment he is purchasing or operating perhaps in the interest of becoming acquainted with maintenance problems. ROBERT S. SMITH

BOOK NOTES

CHEMICAL ENGINEERING IN PRACTICE, edited by James I. Harper. 140 pages, diagrams, 13 × 20 cm. New York, Reinhold Publishing Corp., 1954. Price, $3.00.

Based on a symposium "Chemical Engineering in the Process Industries," this small book should be of interest to all chemists. The symposium was sponsored by the Philadelphia-Wilmington section of the American Institute of Chemical Engineers and the Department of Chemical Engineering at the University of Pennsylvania. The book describes the many phases of industry in which the chemical engineer plays an important role. Eight successful engineers discuss their own fields of process research, process development, process refining, economic analysis, project engineering, construction engineering, operational engineering and market research engineering. The discussions will be of great value to chemical students seeking help in choosing their field of concentration.

A TEXT-BOOK OF RADAR, edited by E. G. Bowen. Second edition, 617 pages, illustrations, 15 × 22 cm. New York, Cambridge University Press, 1954. Price, $8.50.

First appearing in Australia in 1947 and in England in 1948, this comprehensive treatment of radar problems has been brought up-to-date with this second edition. Revisions in the first 512 pages (on basic principles) include corrections of previous errors and additional references. In the last section, the two chapters dealing with the practical applications of radar—military and civil—have been completely rewritten, with emphasis on peacetime uses. A new chapter on application of radar to physical science has been added in this section. The new material includes the application of radar to astronomy, radar aids to surveying, marine and air navigation and the application to meteorology. The work is suitable for graduate and research students and engineers.

MODERN CHEMICAL PROCESSES, VOLUME 3, by the editors of *Industrial & Engineering Chemistry*. 276 pages, illustrations, 21 × 29 cm. New York, Reinhold Publishing Corp., 1954. Price, $5.00.

This is the third volume in a series intended to be a running record of the most modern chemical processes. Volume 3 contains twenty-three recently developed chemical processes, all now in operation on a full scale production level. The descriptions of these processes include a history of the process, a description of the plant installation (including design details), a description of process and operation of plants, the economics of the process (raw materials, plant investment and amortization), personnel, and future prospects. Published every two years, these volumes should prove invaluable as a reference work for practicing industrial chemists.

GRAPHICS IN ENGINEERING AND SCIENCE, by A. S. Levens. 696 pages, diagrams, 18 × 25 cm. New York, John Wiley & Sons, Inc., 1954. Price, $7.00.

This new approach to graphics has as its principal aim "to help the student gain a full appreciation of the graphic language and thus help him . . . so that he can use a powerful mode of expression in analyzing and solving problems that arise in the fields of design, development and research." The book is divided into three parts: (1) Fundamentals and Applications of Orthogonal Projection; (2) Technical Drawing Practices; and (3) Graphical Solutions and Computations. The appendices cover such matters as lettering, geometric constructions, graphical symbols, etc. Profusely illustrated, the book should prove easy to study, and a grasp of its contents will be useful to the engineer in all fields of technology.

ELEMENTS OF MECHANISM, by Venton Levy Doughtie and Walter H. James. 494 pages, diagrams, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1954. Price, $6.00.

The authors give a clear, up-to-date introduction to the design and operation of machinery, based on the classic work of Schwamb, Merrill, James and Doughtie. Like the preceding volumes, the present work is intended to acquaint the reader with the application of fundamental principles of physics in the field of mechanical movement, and to develop in the reader the knowledge needed to analyze any mechanical device. The wealth of problems makes it unnecessary to use a separate work book with the text.

INTRODUCTION TO COLLEGE PHYSICS, by Rogers D. Rusk. 816 pages, illustrations, 16 × 24 cm. New York, Appleton-Century-Crofts, Inc., 1954. Price, $6.50.

The author has written this text for first year college students, either as a terminal course or as a foundation for further study. While the text includes the more recent developments in physics (such as the electron, communications and nuclear physics), the basic classical principles are not neglected. The author's experience as a teacher has enabled him to put together a good, workable text, well-illustrated and with sensible problems.

TABLE OF SECANTS AND COSECANTS TO NINE SIGNIFICANT FIGURES AT HUNDREDTHS OF A DEGREE. National Bureau of Standards Applied Mathematics Series 40. 46 pages, 20 × 26 cm. Washington, Government Printing Office, 1954. Price, $0.35 (paper).

The recent demand for many-figure tables of the trigonometric functions with the argument in decimal division of a degree has prompted this publication. It is anticipated that this table will serve as a useful companion to the previously published table of sines and cosines to fifteen decimal places at hundredths of a degree (Applied Mathematics Series 5). The present table is an outgrowth of the previous volume of sines and cosines. The values were obtained from those of the sine and cosine by division performed on punched-card equipment carrying ten significant figures and rounding to nine. As a partial check, the results were differenced. In the region where this check was inadequate, independent calculations were made and compared with the previous results. As a result of the various tests applied, the entries are considered correct to within six-tenths of a unit in the last place given.

CURRENT TOPICS

Atomic Radiation Used to Sterilize Meats.—Atomic energy can be used to sterilize meats without ill effects on humans, Madison research scientist Dr. Howard Gottlieb said at the University of Wisconsin's Atomic Energy Institute.

Gottlieb, director of the Bjorksten Research Laboratories radiochemical division, spoke at the final session of the three-day institute, sponsored by the UW Extension Division and the College of Engineering.

Gottlieb pointed to three major effects of atomic radiation on food and meats: it kills micro-organisms dangerous to human health, yet has no injurious effects; it sterilizes without heating; and it produces less off-flavor than canning processes. He added that radiation from gamma rays and X-rays will be used for commercial sterilization if economically feasible.

Gottlieb also noted that atomic energy can be used in the manufacture of plastic containers. Radiation improves the curing of plastics, makes the material more flexible, and gives them greater stability in heat. Furthermore, he added, radiation leaves no toxic materials in the bottles.

The three-day institute covered the possibilities of using atomic energy for a wide variety of industrial purposes. Representatives of government, industry, and private research organizations attended.

Dentistry Without Pain?—An ultrasonic tooth drill has been designed by scientists at the Naval Medical Research Institute and proves to be one step nearer to banishing the fear and pain of having tooth cavities filled. The device, which uses ultrasonic waves to do the work of drilling, is a handpiece that can be used easily and comfortably. The heat, noise, vibration and pressure of the present drilling burr have created not only pain for the patient, but, in many people, a fear which it is hoped the new equipment will help to eliminate. While the new device drills a clean hole on extracted teeth, glass and other hard surfaces, it is still in the experimental stage and has not yet been tested on a human patient.

Electronic Clinical Thermometer.—Development of an electronic clinical thermometer has recently been announced by the Army. The new thermometer is said to be more accurate and much faster than the present mercury type, and, according to the Surgeon General of the Army, represents "the first change in clinical thermometers since the mercury column was introduced in 1867." The new device uses metals whose ability to conduct an electric current varies according to temperature. Oxides of the metal are placed in a small metallic tube which is inserted under a patient's tongue or under his arm. The tube is connected by a transmission wire to a small mercury cell battery contained in a plastic case small enough to be held easily in the hand, temperatures being read on a calibrated scale on a meter.

Device to Detect Termites.—There has been developed a listening device so sensitive that it detects a termite's footfall, the crunch of food in its jaws, and the tom-tom-like sounds it

uses to communicate what may be a danger signal to its mates. The instrument consists of a tiny microphone, a hip-pocket power unit and earphones, and promises to make the pest exterminator's job easier when the scientists have completed the job of classifying termite sounds and their causes. The microphone was developed by the Department of Entomology at the University of California in Los Angeles.

Combination Electric Soldering Iron and Temperature Regulator.—Soldering operations in the electronics shop of the U. S. Marine Corps have been improved by the use of a combination soldering iron stand and temperature regulator, suggested by a supervisor. The suggested stand and temperature regulator is simple in design and economical to manufacture. It consists essentially of a metal sleeve equipped with radiating heat dissipators, mounted on a suitable stand. This device does not require other energy dissipators, such as resistances, bulbs, or switches. Damage to this device is practically nonexisting when compared to the damage connected with the use of the conventional type temperature regulators, which require wiring, light bulbs, control switches, etc. In use, it is only necessary for the operator to place the soldering iron in the suggested stand to maintain proper soldering temperature, when the iron is not required for soldering. The radiating heat dissipating design of the device prevents the tip of the iron from reaching such a high temperature that damage occurs. The originating activity reports that this type stand has now been manufactured and supplied to the various shops performing soldering operations, and that the man-hours formerly expended in waiting for the soldering irons to reach proper temperature have been eliminated.

Gas-Tight Seal for Hydrogen Retort.—The initial use of hydrogen retorts used as annealing furnaces at the Naval Ordnance Plant in Indianapolis disclosed that the seals furnished by the manufacturer were not satisfactory and tended to introduce carbon into the atmosphere of the furnace. The original seal was a rope-type material impregnated with oil and graphite. This perplexing problem was finally solved by a leading-man heat treater who designed the seal that prevents leakage. The suggested seal consists of four lengths of 1" × 1" aluminum bar which are welded and machined to fit the seal flange on the face of the retort, and 4 pieces of woven blanket asbestos, the same size as the face of the aluminum seal. Pieces of the woven asbestos are cemented on the face and back of the aluminum seal, in the seal flange on the face of the retort, and on the cover plate. This arrangement provides two layers of asbestos on each side of the aluminum seal; however, the seal is independent of the retort and cover plate and is easily removed to load and unload the furnace.

Inter-City Intrafax System.—The Western Union Telegraph Company has put into operation its first intercity Intrafax system for speeding intracompany communication in facsimile "picture" form. This was accomplished by the linking of twenty banking offices of the County Trust Company, in twelve cities and towns of Westchester County, N. Y., by a 132-mile network. Each banking office is equipped with a compact Intrafax machine over which written, typed or printed matter may be flashed in facsimile simply by pushing a button. All offices on the company's network can immediately examine records and verify customer signatures at the bank's central files in White Plains.

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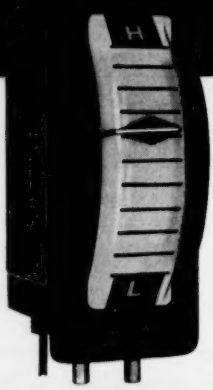

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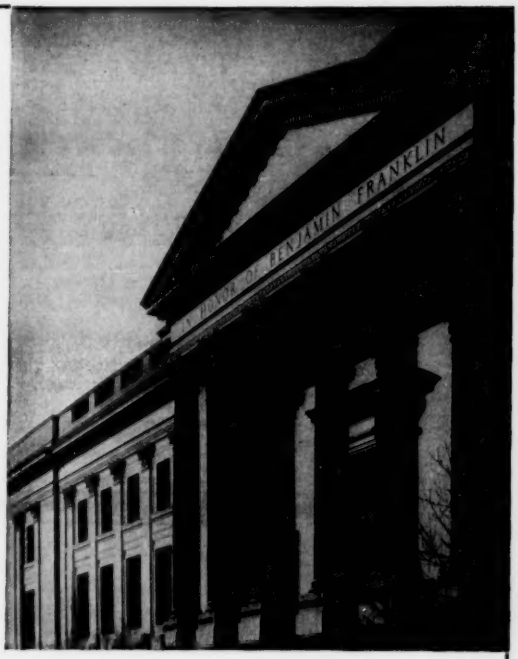

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